Department of Electrical and Electronics Engineering

EEE 415
Power System Analysis I

Transmission Line Parameters

Transmission Line Parameters

Introduction: Overhead vs Underground Transmission

- Overhead Transmission;

  Advantages
  - Cheaper installation cost
  - Lower repair and maintanence costs
  - Better heat dissipation

  Disadvantages
  - Affected by enviromental conditions such as; weather, animals, trees
  - Interference with communication lines
  - Causes visual pollution

  Suitable for long distance high voltage (>36kV) transmission systems.

Transmission Line Parameters

Introduction: Overhead vs Underground Transmission

- Underground Transmission;

  Advantages
  - Lower inductance value, lower voltage drop
  - Reduced interference problems
  - Less affected by enviromental conditions
  - No visual problems

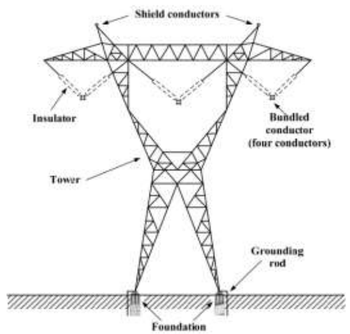
  Disadvantages
  - Higher installation cost; ground needs to excavated and this can be difficult when passing though geographic obstructions such as hills, marshes and rivers.
  - Higher repair and maintanence cost
  - Higher capacitance

  Suitable for short distance subtransimission/distribution systems.

Transmission Line Parameters

Introduction: Construction of Overhead Transmission Lines

- Three-phase conductors, which carry the electric current
- Insulators, which support and electrically isolate the conductors
- Tower, which holds the insulators and conductors
- Foundation and grounding
- Optional shield conductors, which protect against lightening



Transmission Line Parameters

Introduction: Transmission Line Conductors

- Transmission line conductors can be made of copper or aluminum.
- However, aluminum conductors or its alloys have completely replaced copper for overhead transmission because of
  - lower cost
  - lighter weight

of an aluminum conductor compared with a copper conductor of the same current carrying capability.

Transmission Line Parameters

Introduction: Transmission Line Conductors

- Typical transmission line conductors are produced with stranded structure instead of solid structure to increase the tensile strength of conductor.
- Each layer of strands is spiraled in the opposite direction of its adjacent layer to bind strands together.

Transmission Line Parameters

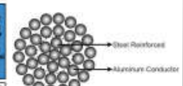
Introduction: Transmission Line Conductors

- The overhead conductor types are;
  - Aluminum Conductor Steel Reinforced (ACSR)
  - All Aluminum Conductor (AAC)
  - All Aluminum Alloy Conductor (AAAC)
  - Aluminum Conductor Alloy Reinforced (ACAR)
  - Aluminum Conductor Steel Supported (ACSS)
  - Gap Type ZT Aluminum Conductor (GTZACSR)
  - Aluminum Conductor Carbon Reinforced (ACFR)
  - Aluminum Conductor Composite Reinforced (ACCR)

Transmission Line Parameters

Tranmission Line Conductors – Sample Data of ACSR Conductors

Code	Size AWG-MCM	Stranding No x Ømm Al.	Stranding Steel	Section mm² Total	Section Al.	Overall Diameter mm Total	Overall Diameter Core	Conductor Weight kg/km Total	Conductor Weight Al.	Conductor Weight Steel	Rated Strength kN	Electrical Resistance 20°C D.C.	Electrical Resistance 25°C A.C.	Electrical Resistance 75°C A.C.	Current Carrying Capacity A
TURKEY	6	6X1,68	1x1,68	15,52	13,3	5,04	1,68	53,8	36,5	17,3	5,295	2,1135	2,1496	2,685	110
SWAN	4	6X2,12	1X2,12	24,71	21,18	6,36	2,12	85,6	58	27,6	8,28	1.3278	1,3537	1,7172	145
SWANATE	4	7X1,96	1x2,61	26,47	21,12	6,53	2,61	99,7	58	41,7	10,5	1.1313	1,3387	1,7383	145
SPARROW	2	6X2,67	1X2,67	39,2	33,6	8,01	2,67	135,9	92,3	43,6	12,68	0.8363	0,8527	1,1081	195
SPARATE	2	7X2,47	1X3,30	42,09	33,54	5,24	3,3	158,8	92,3	66,51	6,2	0.8251	0,8435	1,1181	195
ROBIN	1	6X3,00	1X3,00	49,48	42,41	9	3	171,4	116,4	55	15,8	0.6621	0,6768	0,8907	200
RAVEN	1/0	6X3,37	1X3,37	62,44	53,52	10,11	3,37	216,1	146,7	69,6	19,69	0.5243	0,5370	0,7145	255
QUAIL	2/0	6X3,78	1X3,78	78,55	67,33	11,34	3,78	272,5	185	87,5	23,63	0.416	0,4265	0,5803	295
PIGEON	3/0	6X4,25	1x4,25	99,31	85,12	12,75	4,25	343,5	233,2	110,3	29,46	0.3304	0,3386	0,4705	340
PENGUIN	4/0	6X4,77	1X4,77	125,09	107,22	14,31	4,77	433,2	294,2	139	37,16	0.2618	0,2697	0,3829	390
WAXWING	266,8	18X3,09	1X3,09	142,5	135	15,45	3,09	431,6	372,9	58,7	30,62	0.2119	0,2169	0,2595	480
PARTRIDGE	266,8	26X2,57	7X2,00	156,9	134,9	16,28	6	546,1	374,3	171,8	50,28	0.21	0,2146	0,2569	490
OSTRICH	300	26X2,73	7X2,12	176,9	152,2	17,28	6,36	614,6	421,3	193,3	56,52	0.1867	0,1909	0,2283	530
MERLIN	336,4	18X3,47	1X3,47	179,7	170,2	17,35	3,47	543,2	469,7	73,5	38,36	0.168	0,1719	0,2057	560
LINNET	336,4	26X2,89	7X2,25	198,4	170,6	18,31	6,75	689	472,2	216,8	62,75	0.1663	0,1669	0,2037	570
ORIOLE	336,4	30X2,69	7X2,69	210,3	170,5	18,83	8,07	784,3	473,2	311,1	76,98	0.1654	0,169	0,2024	575
CHICKADEE	397,5	18X3,77	1X3,77	212,1	200,9	18,85	3,77	642,9	555,5	87,4	44,23	0.1221	0,1457	0,1742	620
BRANT	397,5	24X3,27	7X2,18	227,7	201,6	19,62	6,54	762	558,1	203,9	64,97	0.1417	0,145	0,1732	630
IBIS	397,5	26X3,14	7X2,44	234	201,3	19,88	7,32	814,0	558,2	255,8	72,53	0.1411	0,1444	0,1726	635
LARK	397,5	30X2,92	7X2,92	247,8	200,9	20,44	8,76	927,1	555,1	372	90,3	0.1401	0,1434	0,1716	645
PELICAN	477	18X4,14	1X4,14	255,8	242,3	20,7	4,14	770,9	666,6	104,5	52,51	0.1184	0,1217	0,1453	700
FLICKER	477	24X3,58	7X2,39	273	241,6	21,19	7,17	915,2	670,1	245,1	74,54	0.1178	0,1207	0,1444	710
HAWK	477	26X3,44	7X2,67	280,8	241,6	21,77	8,01	977,7	669,7	308	86,77	0.1171	0,1201	0,1437	715
HEN	477	30X3,20	7X3,20	297,6	241,3	22,4	9,6	1.111,7	671	440,7	105,91	0.1165	0,1194	0,1427	725



Transmission Line Parameters

Tranmission Line Conductors – Sample Data of ACSR Conductors

ÇELİK ÖZLÜ ALÜMİNYUM İLETKENLERİN KARAKTERİSTİKLERİ							
İLETKEN ACSR	Anma Kesit MCM		477	795	954	954	1272
	Kod İsmi		HAWK	DRAKE	RAIL	CARDINAL	PHEASANT
STANDART KESİT(mm²)	Alüminyum		241,65	402,56	484,40	484,53	645,08
	Çelik		39,19	65,44	33,60	62,81	81,71
	Toplam		280,84	468,00	517,00	547,34	726,79
KATMAN SAYISI			3	3	4	4	5
KATMANLARDAKİ TEL SAYISI			1/6/10/16	1/6/10/16	1/6/9/15/21	1/6/12/18/24	1/6/12/12/18/24
ÖRGÜDEKİ ALÜMİNYUM TELİN		Adedi	26	26	45	54	54
		Çapı(mm)	3,44	4,44	3,70	3,38	3,90
ÖRGÜDEKİ ÇELİK TELİN		Adedi	7	7	7	7	19
		Çapı(mm)	2,67	3,45	2,48	3,38	2,34
STANDART ÇAP(mm)	Komple İletken		21,77	28,11	29,60	30,42	35,10
	Çelik Nüve		8,01	10,36	7,4	10,14	11,70
ANMA KOPMA KUVVETİ(kgf)			8798	14165	11864	15589	20357
BİRİM AĞIRLIK (kg/km)			973	1624	1600,2	1829	2423,5
NİHAİ ELASTİSİTE MODÜLÜ (kg/mm²)			7700	7700	6700	7000	6800
ORTALAMA ISIL UZAMA KATSAYISI ($1/^0C \times 10^{-6}$)			18,9	18,9	19,5	19,3	19,4
BİR MAKARADA ALÜMİNYUM TELLERDEKİ İZİN VERİLEN EKLERİN SAYISI			4	4	5	5	5

Transmission Line Parameters

Tranmission Line Conductors – Sample Data of AAC Conductors

Code Word / Kod	Size / Boyut AWG-MCM	Stranding / Kompozisyon N2 x Ø mm	Section / Kesit mm²	Overall Diameter / Toplam Çap mm	Cable Weight / Birim Ağırlık kg/km	Rated Strength / Kopma Mukavemeti N	Electrical Resistance / Elektrik Direnci: DC. 20 °C Ω/km	A.C.: CC. 25 °C Ω/km	A.C.: CC. 25 °C Ω/km	Current Carrying Capacity / Akım Taşıma Kapasitesi (1) A
PEACHBELL	6	7X1,56	13,21	4,65	36,6	2,5	1,1702	2,2129	2,6499	110
ROSE	4	7X1,96	21,12	5,88	58,3	3,92	1,3638	1,3914	1,6663	145
IRIS	2	7X2,47	33,54	7,41	92,7	6,01	0,857	0,855	1,0486	195
PANSY	1	7X2,78	42,69	8,34	116,8	7,3	0,6801	0,6962	0,8310	225
POPPY	1/0	7X3,12	53,52	9,36	147,5	8,86	0,539	0,5499	0,6588	260
ASTER	2/0	7X3,50	67,34	10,5	185,9	11,17	0,4275	0,4374	0,5226	305
PHLOX	3/0	7X3,93	84,91	11,79	234,4	13,35	0,3392	0,3468	0,415	350
OXLIP	4/0	7X4,42	107,4	13,26	295,6	17,05	0,2689	0,2747	0,3288	410
SNEEZEWORTH	250	7X4,80	127,6	14,4	349,3	20,12	0,2273	0,2324	0,2784	455
VALERIAN	250	19X2,91	126,4	14,55	349,3	20,74	0,2273	0,2324	0,2784	455
DAISY	266,8	7X4,96	135,3	14,88	327,8	21,5	0,2133	0,2181	0,261	475
LAUREL	266,8	19X3,01	135,2	15,05	327,8	22,12	0,2133	0,2181	0,261	475
PEONY	300	19X3,19	151,9	15,95	419,1	24,38	0,1897	0,1945	0,2324	515
TULIP	336,4	19X3,38	170,5	16,9	470	27,37	0,1691	0,1734	0,2074	555
DAFFODIL	350	19X3,45	177,6	17,25	489	28,45	0,1626	0,1666	0,1995	565
CANNA	397,5	19X3,68	202,1	18,4	556,4	31,64	0,1431	0,1473	0,1659	615
GOLDENTUFT	450	19X3,91	228,1	19,55	682,6	35,11	0,1264	0,1299	0,156	665
COSMOS	477	19X4,02	241,2	20,1	666,4	37,2	0,1193	0,1224	0,1467	690
SYRINGA	477	37X2,88	241	20,16	666,4	38,67	0,1193	0,1224	0,1467	690
ZINNIA	500	19X4,12	253,3	20,6	698,6	38,98	0,1138	0,1168	0,1398	715
HYACINTH	500	37X2,95	252,9	20,65	698,6	40,54	0,1138	0,1168	0,1398	715
DAHLIA	556,5	19X4,35	282,4	21,75	777,4	43,39	0,1022	0,1089	0,1262	765
MISTLETOE	556,5	37X3,11	281,1	21,77	777,4	44,25	0,1022	0,1089	0,1262	765

Transmission Line Parameters

Tranmission Line Conductors – Sample Data of AAC Conductors

		TAM ALUMİNYUM İLETKENLER						
		1	2	3	4	5	6	7
		ROSE	LILY	PANSY	POPPY	ASTER	PHLOX	OXLIP
AWG VEYA SİRKÜLER MİL KESİTİ		4	3	1	1/0	2/0	3/0	4/0
KESİT ALANI	AL. mm2	21.14	26.60	42.49	53.48	67.14	84.91	107.38
	ST mm2	-	-	-	-	-	-	-
	TOPLAM mm2	21.14	26.60	42.49	53.48	67.14	84.91	107.38
TEL SAYISI VE ÇAPI	ad.	7	7	7	7	7	7	7
	AL. Mm2	1.96	2.20	2.78	3.12	3.50	3.93	4.42
	ad.	-	-	-	-	-	-	-
	ST mm2	-	-	-	-	-	-	-
ÇAP	ÇELİK ÖZ mm	-	-	-	-	-	-	-
	TOPLAM mm	5.88	6.6	8.34	9.36	10.5	11.79	13.26
ANMA KOPMA YÜKÜ kg		403	495	725	888	1115	1369	1732
DA DİRENCİ, 20 0C ohm/km.		1.3558	1.0766	0.6743	0.5354	0.4254	0.3372	0.2662
BİRİM AĞIRLIK	AL. kg/km	57.8	72.8	116.4	146.4	184.4	232.5	294
	ST kg/km	-	-	-	-	-	-	-
	TOPLAM kg/km	57.8	72.8	116.4	146.4	184.4	232.5	294
MAKARA BAŞINA TAVSİYE EDİLEN İLETKEN BOYU m		9500	7500	4800	3800	3000	2400	1900

Transmission Line Parameters

Introduction: Bundled Conductors

- In EHV (>230kV) transmission lines, the surface potential gradient can exceed the dielectric strength of the surrounding air. This is called as corona discharge.
- The corona discharge causes the ionization of air which surrounds the conductors. The corona will occur when the strength (potential gradient) of the electric field around a conductor is high enough to form a conductive region, but not high enough to cause electrical breakdown or arcing to nearby objects.
- The corona discharge can cause significant power loss and interference with communication circuits.
- In order to reduce corona discharge in transmission lines, the transmission lines are formed from more than one conductor per phase which is called **Bundled Conductors**.

Transmission Line Parameters

Introduction: Bundled Conductors

- Bundled conductors have several advantages:
  - Prevent corona discharge
  - Increase effective diameter of conductors so,
    - The inductances of lines are decreased.
    - The capacitances of lines are increased.
    - The voltage drops on lines are decreased.
    - The power transfer capabilities of lines are increased.

Transmission Line Parameters

Introduction: British & North American Units

- 1 feet (ft) = 30.48 cm 1ft = 12in
- 1 inch(in) = 2.54 cm
- 1 mil = 0.001 inch = 0.0254 mm
- Circular Mil: A circular mil is a unit of area, equal to the area of a circle with a diameter of one mil (one thousandth of an inch).
  1 cmil = 0.0005067 mm²
- MCM: Mega Circular Miles
  1 MCM = 1000 cmil = 1 kcmil = 0.5067 mm²

Transmission Line Parameters

Introduction: British & North American Units

- AWG = American Wire Gauge

The diameter of a No. *n* AWG wire is determined, for gauges smaller than 00 (36 to 0), according to the following formula:

$$d_n = 0.005\ \text{inch} \times 92^{\frac{36-n}{39}} = 0.127\ \text{mm} \times 92^{\frac{36-n}{39}}$$

Sizes with multiple zeros are successively larger than No. 0 and can be denoted using "*number of zeros*/0", for example 4/0 for 0000. For an *m*/0 AWG wire, use *n* = (*m* 1) = 1 *m* in the above formulas. For instance, for No. 0000 or 4/0, use n = 3.

AWG	Diameter		Turns of wire, without insulation		Area	
	(in)	(mm)	(per in)	(per cm)	(kcmil)	(mm²)
0000 (4/0)	0.4600	11.684	2.17	0.856	212	107
000 (3/0)	0.4096	10.405	2.44	0.961	168	85.0
00 (2/0)	0.3648	9.266	2.74	1.08	133	67.4
0 (1/0)	0.3249	8.251	3.08	1.21	106	53.5
1	0.2893	7.348	3.46	1.36	83.7	42.4
2	0.2576	6.544	3.88	1.53	66.4	33.6
3	0.2294	5.827	4.36	1.72	52.6	26.7
4	0.2043	5.189	4.89	1.93	41.7	21.2

Transmission Line Parameters

Parameters of Transmission Lines

- Resistance (R)
- Conductance (G)
- Inductance (L)
- Capacitance (C)

Transmission Line Parameters
Resistance

- The dc resistance of a conductor at a specified temperature T is

$$R_{dc,T} = \frac{\rho_T l}{A} \quad \Omega$$

where

ρ_T = conductor resistivity at temperature T
l = conductor length
A = conductor cross-sectional area

Transmission Line Parameters
Resistance

- Conductor resistance depends on the following factors:
  - Spiraling
  - Temperature
  - Frequency – Skin Effect

Transmission Line Parameters
Resistance

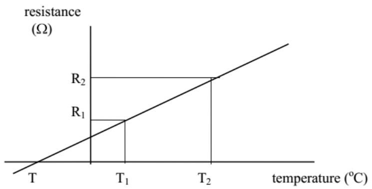
- Spiraling
  - For stranded conductors, alternate layers of strands are spiraled in opposite directions to hold the strands together.
  - Spiraling makes the strands 1 or 2% longer than the actual conductor length.
  - As a result, the dc resistance of a stranded conductor is 1 or 2% larger than that calculated value for a specified conductor length.

Transmission Line Parameters
Resistance

- Temperature
  - Resistivity of conductor metals varies linearly over normal operating temperatures according to

$$\rho_{T2} = \rho_{T1}\left(\frac{T_2 + T}{T_1 + T}\right)$$

where ρ_{T1} and ρ_{T2} are resistivities at temperatures T_1 and T_2°C, respectively. T is a temperature constant that depends on the conductor material.



Transmission Line Parameters
Resistance

- Temperature

Material	% Conductivity	$\rho_{20°C}$ Resistivity at 20 °C $\Omega m \times 10^{-8}$	T Temperature Constant °C
Copper:			
Annealed	100%	1.72	234.5
Hard-drawn	97.3%	1.77	241.5
Aluminum			
Hard-drawn	61%	2.83	228.1
Brass	20–27%	6.4–8.4	480
Iron	17.2%	10	180
Silver	108%	1.59	243
Sodium	40%	4.3	207
Steel	2–14%	12–88	180–980

Transmission Line Parameters
Resistance

- Frequency – Skin Effect
  - For dc, the current distribution is uniform throughout the conductor cross section. However, for ac, the current distribution is non-uniform. As frequency increases, the current in a solid cylindrical conductor tends to crowd toward the conductor surface, with smaller current density at the conductor center. This phenomenon is called ***skin effect***.
  - The skin effect causes the decrease of effective area of conductors. Thus, the ac resistance of conductor is higher than the dc resistance.

Transmission Line Parameters
Resistance

- The ac resistance or effective resistance of a conductor can be calculated using,

$$R_{ac} = \frac{P_{loss}}{|I|^2} \quad \Omega$$

where P_{loss} is the conductor real power loss in watts and I is the rms conductor current.

- Wire manufacturers usually supply tables of resistance per unit length at common frequencies (50 or 60 Hz) and different temperatures. Therefore, the resistance can be determined from such tables.

Transmission Line Parameters
Conductance

- Conductance accounts for real power loss between conductors or between conductors and ground. For overhead lines, this power loss is due to leakage currents at insulators and to corona.
- Insulator leakage current depends on the amount of dirt, salt, and other contaminants that have accumulated on insulators, as well as on meteorological factors, particularly the presence of moisture.
- Corona occurs when a high value of electric field strength at a conductor surface causes the air to become electrically ionized and to conduct. The real power loss due to corona, called corona loss, depends on meteorological conditions, particularly rain, and on conductor surface irregularities.
- Losses due to insulator leakage and corona are usually small compared to conductor I^2R loss. Conductance is usually neglected in power system studies because it is a very small component of the shunt admittance.

Transmission Line Parameters
Inductance: Solid Cylindrical Conductor

- The inductance of a magnetic circuit that has a constant permeability μ can be obtained by determining the following:

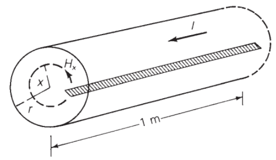
  1. Magnetic field intensity H, from Ampere's law.
  2. Magnetic flux density B. (B = μH)
  3. Flux linkages λ.
  4. Inductance from flux linkages per ampere (L = λI)

Transmission Line Parameters
Inductance: Solid Cylindrical Conductor

Internal Inductance (Self Inductance)

For simplicity, assume that the conductor;

(1) is sufficiently long that end effects are neglected.
(2) is nonmagnetic ($\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m).
(3) has a uniform current density (skin effect is neglected).



Transmission Line Parameters
Inductance: Solid Cylindrical Conductor

Internal Inductance (Self Inductance)

- From Ampere's law;

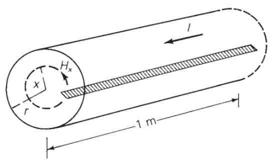
$$\oint H_{\tan}\, dl = I_{\text{enclosed}}$$

- To determine the magnetic field inside the conductor, select the dashed circle of radius x < r.

$$H_x(2\pi x) = I_x \qquad \text{for } x < r$$

$$H_x = \frac{I_x}{2\pi x} \quad \text{A/m}$$

where I_x is the portion of the total current enclosed by the contour.



Transmission Line Parameters
Inductance: Solid Cylindrical Conductor

Internal Inductance (Self Inductance)

- Assume that a uniform current distribution within the conductor, the current enclosed by the conductor is equal to:

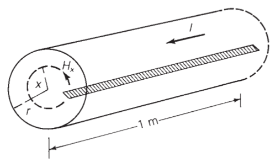
$$I_x = \left(\frac{x}{r}\right)^2 I \qquad \text{for } x < r$$

- The magnetic field inside the conductor becomes;

$$H_x = \frac{xI}{2\pi r^2} \quad \text{A/m}$$

- For a nonmagnetic conductor, the magnetic flux density B_x is

$$B_x = \mu_0 H_x = \frac{\mu_0 x I}{2\pi r^2} \quad \text{Wb/m}^2$$



Transmission Line Parameters

Inductance: Solid Cylindrical Conductor

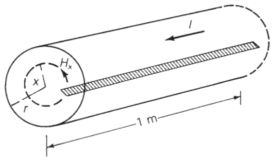
Internal Inductance

- The differential flux $d\varphi$ per-unit length of conductor in the cross-hatched rectangle of width dx is;

$$d\Phi = B_x\, dx \quad \text{Wb/m}$$

- Since only the fraction $(x/r)^2$ of the total current I is linked by the flux, the computation of the differential flux linkage $d\lambda$ in the rectangle is

$$d\lambda = \left(\frac{x}{r}\right)^2 d\Phi = \frac{\mu_0 I}{2\pi r^4} x^3\, dx \quad \text{Wb-t/m}$$



Transmission Line Parameters

Inductance: Solid Cylindrical Conductor

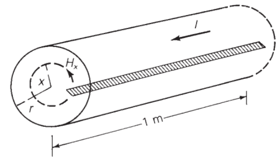
Internal Inductance

- λ_{int} inside the conductor is determined by integrating $d\lambda$ from $x=0$ to $x=r$ as;

$$\lambda_{\text{int}} = \int_0^r d\lambda = \frac{\mu_0 I}{2\pi r^4}\int_0^r x^3\, dx = \frac{\mu_0 I}{8\pi} = \frac{1}{2}\times 10^{-7} I \quad \text{Wb-t/m}$$

- The internal inductance Lint per-unit length of conductor due to this flux linkage is;

$$\text{L}_{\text{int}} = \frac{\lambda_{\text{int}}}{I} = \frac{\mu_0}{8\pi} = \frac{1}{2}\times 10^{-7} \quad \text{H/m}$$



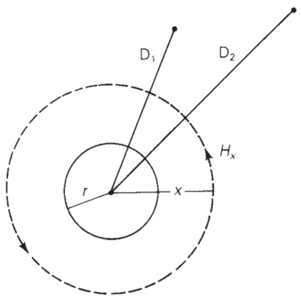
Transmission Line Parameters

Inductance: Solid Cylindrical Conductor

External Inductance

- To determine the magnetic field outside the conductor, select the dashed circle of radius x > r as the closed contour for Ampere's law.
- In this condition, this contour encloses the entire current I so,

$$H_x = \frac{I}{2\pi x} \quad \text{A/m} \qquad x > r$$



Transmission Line Parameters

Inductance: Solid Cylindrical Conductor

External Inductance

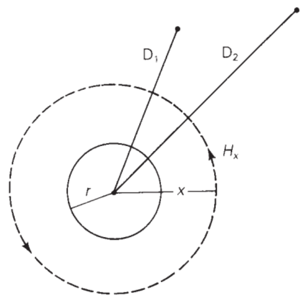
- Outside the conductor, $\mu = \mu_0$ so,

$$B_x = \mu_0 H_x = (4\pi \times 10^{-7})\frac{I}{2\pi x} = 2\times 10^{-7}\frac{I}{x} \quad \text{Wb/m}^2$$

$$d\Phi = B_x\, dx = 2\times 10^{-7}\frac{I}{x}\, dx \quad \text{Wb/m}$$

- Since the entire current I is linked by the flux outside the conductor,

$$d\lambda = d\Phi = 2\times 10^{-7}\frac{I}{x}\, dx \quad \text{Wb-t/m}$$



Transmission Line Parameters
Inductance: Solid Cylindrical Conductor

External Inductance

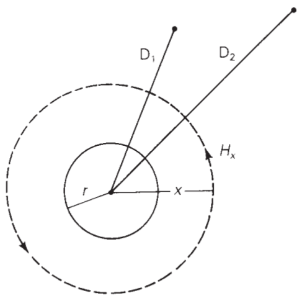
- Integrating $d\lambda$ between two external points at distances D_1 and D_2 from the conductor center gives the external flux linkage λ_{12} between D_1 and D_2:

$$\lambda_{12} = \int_{D_1}^{D_2} d\lambda = 2 \times 10^{-7} I \int_{D_1}^{D_2} \frac{dx}{x}$$

$$= 2 \times 10^{-7} I \ln\left(\frac{D_2}{D_1}\right) \quad \text{Wb-t/m}$$

- The external inductance L_{12} per-unit length due to the flux linkages between D_1 and D_2 is then

$$L_{12} = \frac{\lambda_{12}}{I} = 2 \times 10^{-7} \ln\left(\frac{D_2}{D_1}\right) \quad \text{H/m}$$



Transmission Line Parameters
Inductance: Solid Cylindrical Conductor

Total Inductance

- The total flux λ_P linking the conductor out to external point P at distance D is the sum of the internal flux linkage and the external flux linkage from $D_1 = r$ to $D_2 = D$ as,

$$\lambda_P = \frac{1}{2} \times 10^{-7} I + 2 \times 10^{-7} I \ln \frac{D}{r}$$

- Using the identity ½ = 2*ln* $e^{1/4}$ a more convenient expression for λ_P is obtained:

$$\lambda_P = 2 \times 10^{-7} I \left(\ln e^{1/4} + \ln \frac{D}{r}\right)$$

$$= 2 \times 10^{-7} I \ln \frac{D}{e^{-1/4} r}$$

$$= 2 \times 10^{-7} I \ln \frac{D}{r'} \quad \text{Wb-t/m} \qquad r' = e^{-1/4} r = 0.7788r$$

Transmission Line Parameters
Inductance: Solid Cylindrical Conductor

Total Inductance

- The total inductance L_P due to both internal and external flux linkages out to distance D is;

$$L_P = \frac{\lambda_P}{I} = 2 \times 10^{-7} \ln\left(\frac{D}{r'}\right) \quad \text{H/m}$$

Transmission Line Parameters
Inductance: Solid Cylindrical Conductor

The Array of Solid Cylindrical Conductors

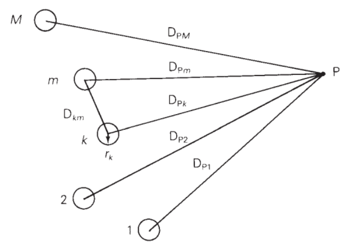
- Assume that each conductor m carries current I_m and the sum of the conductor currents is zero as,

$$I_1 + I_2 + \cdots + I_M = \sum_{m=1}^{M} I_m = 0$$

- The flux linkage λ_{kPk} which links conductor k out to point P due to current λ_k is,

$$\lambda_{kPk} = 2 \times 10^{-7} I_k \ln \frac{D_{Pk}}{r_k'}$$

- λ_{kPk} includes both internal and external flux linkages due to I_k.



Transmission Line Parameters

Inductance: Solid Cylindrical Conductor

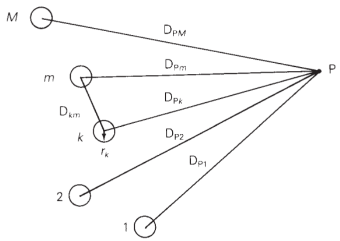
The Array of Solid Cylindrical Conductors

- The flux linkage λ_{kPm}, which links conductor k out to P due to I_m is,

$$\lambda_{kPm} = 2 \times 10^{-7} I_m \ln \frac{D_{Pm}}{D_{km}}$$

$D_{km} >> r_k$

$D_{km} \approx (D_{km} - r_k) \approx (D_{km} + r_k)$



Transmission Line Parameters

Inductance: Solid Cylindrical Conductor

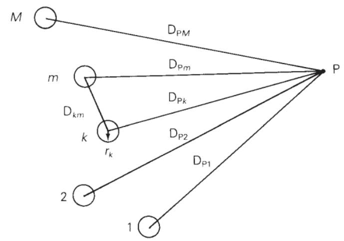
The Array of Solid Cylindrical Conductors

- Using superposition, the total flux linkage λ_{kP}, which links conductor k out to P due to all the currents is,

$$\lambda_{kP} = \lambda_{kP1} + \lambda_{kP2} + \cdots + \lambda_{kPM}$$

$$= 2 \times 10^{-7} \sum_{m=1}^{M} I_m \ln \frac{D_{Pm}}{D_{km}}$$

where we define $D_{kk} = r'_k = e^{-1/4} r_k$ when $m = k$ in the above summation.



Transmission Line Parameters

Inductance: Solid Cylindrical Conductor

The Array of Solid Cylindrical Conductors

$$\lambda_{kP} = \lambda_{kP1} + \lambda_{kP2} + \cdots + \lambda_{kPM}$$

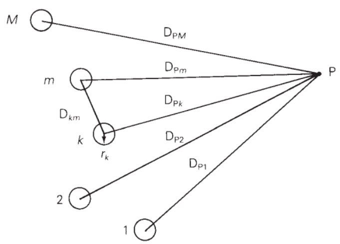
$$= 2 \times 10^{-7} \sum_{m=1}^{M} I_m \ln \frac{D_{Pm}}{D_{km}}$$

- The above equation can be separated into two summations as;

$$\lambda_{kP} = 2 \times 10^{-7} \sum_{m=1}^{M} I_m \ln \frac{1}{D_{km}} + 2 \times 10^{-7} \sum_{m=1}^{M} I_m \ln D_{Pm}$$

- Removing the last term from the second summation we can get;

$$\lambda_{kP} = 2 \times 10^{-7} \left[\sum_{m=1}^{M} I_m \ln \frac{1}{D_{km}} + \sum_{m=1}^{M-1} I_m \ln D_{Pm} + I_M \ln D_{PM} \right]$$



Transmission Line Parameters

Inductance: Solid Cylindrical Conductor

The Array of Solid Cylindrical Conductors

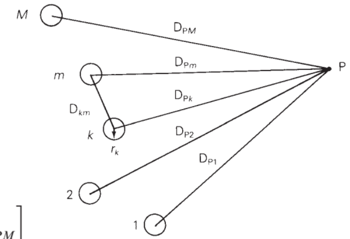
$$\lambda_{kP} = 2 \times 10^{-7} \left[\sum_{m=1}^{M} I_m \ln \frac{1}{D_{km}} + \sum_{m=1}^{M-1} I_m \ln D_{Pm} + I_M \ln D_{PM} \right]$$

$$I_1 + I_2 + \cdots + I_M = \sum_{m=1}^{M} I_m = 0$$

$$I_M = -(I_1 + I_2 + \cdots + I_{M-1}) = -\sum_{m=1}^{M-1} I_m$$

$$\lambda_{kP} = 2 \times 10^{-7} \left[\sum_{m=1}^{M} I_m \ln \frac{1}{D_{km}} + \sum_{m=1}^{M-1} I_m \ln D_{Pm} - \sum_{m=1}^{M-1} I_m \ln D_{PM} \right]$$

$$= 2 \times 10^{-7} \left[\sum_{m=1}^{M} I_m \ln \frac{1}{D_{km}} + \sum_{m=1}^{M-1} I_m \ln \frac{D_{Pm}}{D_{PM}} \right] \quad ($$



Transmission Line Parameters

Inductance: Solid Cylindrical Conductor

The Array of Solid Cylindrical Conductors

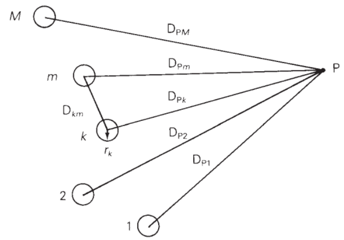
$$\lambda_{kP} = 2\times 10^{-7}\left[\sum_{m=1}^{M} I_m \ln\frac{1}{D_{km}} + \sum_{m=1}^{M-1} I_m \ln D_{Pm} - \sum_{m=1}^{M-1} I_m \ln D_{PM}\right]$$

$$= 2\times 10^{-7}\left[\sum_{m=1}^{M} I_m \ln\frac{1}{D_{km}} + \sum_{m=1}^{M-1} I_m \ln\frac{D_{Pm}}{D_{PM}}\right]$$

- When $P \rightarrow \infty$, all the distances D_{Pm} become equal, the ratios D_{Pm}=D_{PM} become unity, and $\ln(D_{Pm}/D_{PM}) \rightarrow 0$.
- The total flux linking conductor k out to infinity λ_k is equal to;

$$\lambda_k = \lim_{P\to\infty} \lambda_{kP}$$

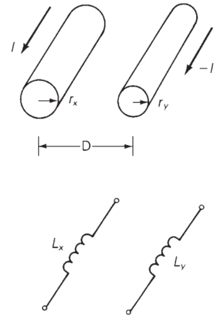
$$\lambda_k = 2\times 10^{-7}\sum_{m=1}^{M} I_m \ln\frac{1}{D_{km}} \quad \text{Wb-t/m}$$



Transmission Line Parameters

Inductance: Single Phase Two Wire Line

- Conductor x with radius r_x carries phasor current $I_x = I$ referenced out of the page.
- Conductor y with radius r_y carries return current $I_y = -I$.
- The sum of the two currents is zero.



Transmission Line Parameters

Inductance: Single Phase Two Wire Line

- The total flux linking conductor x is;

$$\lambda_x = 2\times 10^{-7}\left(I_x \ln\frac{1}{D_{xx}} + I_y \ln\frac{1}{D_{xy}}\right)$$

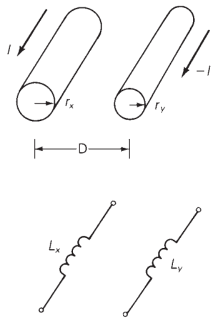
$$= 2\times 10^{-7}\left(I \ln\frac{1}{r'_x} - I \ln\frac{1}{D}\right)$$

$$= 2\times 10^{-7} I \ln\frac{D}{r'_x} \quad \text{Wb-t/m}$$

where $r'_x = e^{-1/4} r_x = 0.7788 r_x$

- The inductance of conductor x is;

$$L_x = \frac{\lambda_x}{I_x} = \frac{\lambda_x}{I} = 2\times 10^{-7}\ln\frac{D}{r'_x} \quad \text{H/m per conductor}$$



Transmission Line Parameters

Inductance: Single Phase Two Wire Line

- Similarly, the total flux linking conductor y is

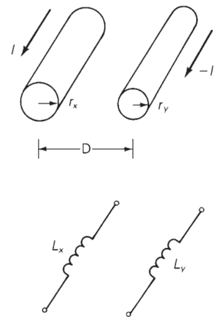
$$\lambda_y = 2\times 10^{-7}\left(I_x \ln\frac{1}{D_{yx}} + I_y \ln\frac{1}{D_{yy}}\right)$$

$$= 2\times 10^{-7}\left(I \ln\frac{1}{D} - I \ln\frac{1}{r'_y}\right)$$

$$= -2\times 10^{-7} I \ln\frac{D}{r'_y}$$

- The inductance of conductor y is;

$$L_y = \frac{\lambda_y}{I_y} = \frac{\lambda_y}{-I} = 2\times 10^{-7}\ln\frac{D}{r'_y} \quad \text{H/m per conductor}$$



Transmission Line Parameters

Inductance: Single Phase Two Wire Line

- The total inductance of the single-phase circuit, also called loop inductance, is;

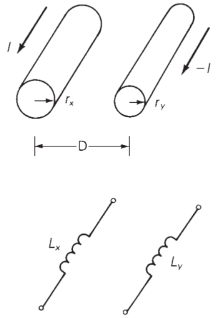
$$L = L_x + L_y = 2\times10^{-7}\left(\ln\frac{D}{r'_x} + \ln\frac{D}{r'_y}\right)$$

$$= 2\times10^{-7}\ln\frac{D^2}{r'_x r'_y}$$

$$= 4\times10^{-7}\ln\frac{D}{\sqrt{r'_x r'_y}} \quad \text{H/m per circuit}$$

if $r'_x = r'_y = r'$, the total circuit inductance is

$$L = 4\times10^{-7}\ln\frac{D}{r'} \quad \text{H/m per circuit}$$



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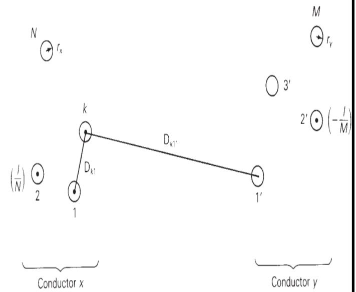
Transmission Line Parameters

Inductance: Composite Conductors and Unequal Spacing

- The composite conductors consist of two or more solid cylindrical subconductors in parallel.
- A stranded conductor is one example of a composite conductor.
- For simplicity of calculations, it is assumed that the subconductors are identical and share the conductor current equally for each conductor.



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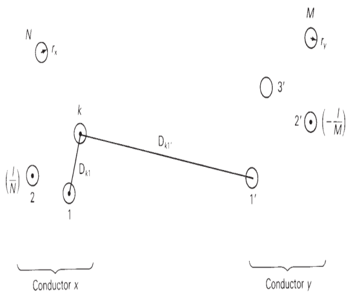
Transmission Line Parameters

Inductance: Composite Conductors and Unequal Spacing

- A single-phase two-conductor line consisting of two composite conductors *x* and *y*.
- Conductor *x* has N identical subconductors, each with radius r_x and with current *(I/N)* referenced out of the page.
- Similarly, conductor *y* consists of M identical subconductors, each with radius r_y and with return current *(-I/M)*.



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Transmission Line Parameters

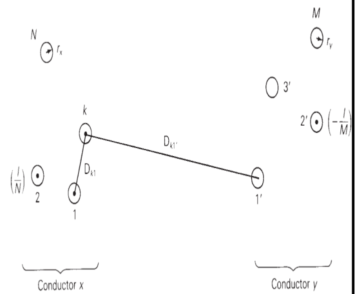
Inductance: Composite Conductors and Unequal Spacing

- Since the sum of all the currents is zero, the total flux φ_k linking subconductor *k* of conductor *x* is;

$$\Phi_k = 2\times10^{-7}\left[\frac{I}{N}\sum_{m=1}^{N}\ln\frac{1}{D_{km}} - \frac{I}{M}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}\right]$$

- Since only the fraction *(1 /N)* of the total conductor current I is linked by this flux, the flux linkage λ_k of (the current in) subconductor *k* is

$$\lambda_k = \frac{\Phi_k}{N} = 2\times10^{-7} I\left[\frac{1}{N^2}\sum_{m=1}^{N}\ln\frac{1}{D_{km}} - \frac{1}{NM}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}\right]$$



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Transmission Line Parameters

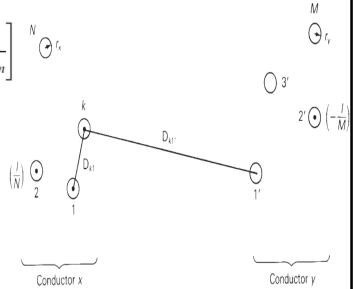
Inductance: Composite Conductors and Unequal Spacing

- The total flux linkage of conductor x is

$$\lambda_x = \sum_{k=1}^{N} \lambda_k = 2 \times 10^{-7} I \sum_{k=1}^{N} \left[\frac{1}{N^2} \sum_{m=1}^{N} \ln \frac{1}{D_{km}} - \frac{1}{NM} \sum_{m=1'}^{M} \ln \frac{1}{D_{km}} \right]$$

- Using $\ln A^{\alpha} = \alpha \ln A$ and $\sum \ln A_k = \ln \prod A_k$
- The total flux linkage of conductor x can reorginized as;

$$\lambda_x = 2 \times 10^{-7} I \ln \prod_{k=1}^{N} \frac{\left(\prod_{m=1'}^{M} D_{km} \right)^{1/NM}}{\left(\prod_{m=1}^{N} D_{km} \right)^{1/N^2}}$$



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Transmission Line Parameters

Inductance: Composite Conductors and Unequal Spacing

- The total flux linkage of conductor x is

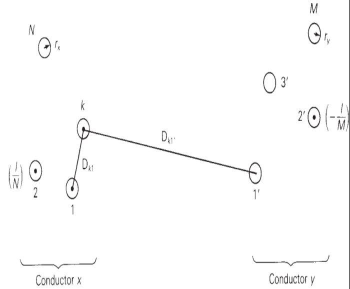
$$L_x = \frac{\lambda_x}{I}$$

$$L_x = 2 \times 10^{-7} \ln \frac{D_{xy}}{D_{xx}} \quad \text{H/m per conductor}$$

where

$$D_{xy} = \sqrt[MN]{\prod_{k=1}^{N} \prod_{m=1'}^{M} D_{km}}$$

$$D_{xx} = \sqrt[N^2]{\prod_{k=1}^{N} \prod_{m=1}^{N} D_{km}}$$



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Transmission Line Parameters

Inductance: Composite Conductors and Unequal Spacing

- D_{xy} is called the **geometric mean distance** or **GMD** between conductors *x* and *y*.
- D_{xx} is called the **geometric mean radius** or **GMR** of conductor *x*.
- Similarly, for conductor y,

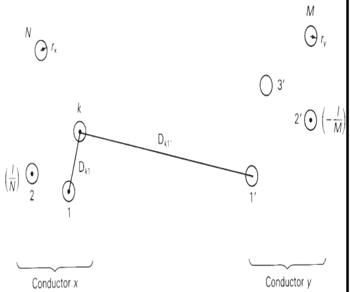
$$L_y = 2 \times 10^{-7} \ln \frac{D_{xy}}{D_{yy}} \quad \text{H/m per conductor}$$

- D_{yy}, the GMR of conductor *y*, is

$$D_{yy} = \sqrt[M^2]{\prod_{k=1'}^{M} \prod_{m=1'}^{M} D_{km}}$$

- The total inductance L of the single-phase circuit is

$$L = L_x + L_y \quad \text{H/m per circuit}$$



Transmission Line Parameters

Inductance: Composite Conductors and Unequal Spacing

- It is seldom necessary to calculate GMR or GMD for standard lines.
- The GMR of standard conductors is provided by conductor manufacturers and can be found in various handbooks.
- Also, if the distances between conductors are large compared to the distances between subconductors of each conductor, then the GMD between conductors is approximately equal to the distance between conductor centers.

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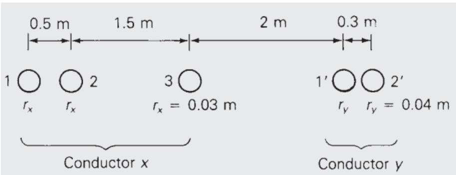
Transmission Line Parameters

Inductance: Composite Conductors and Unequal Spacing

- **Example 1:**

Evaluate the inductance of conductor x, conductor y and the total inductance in H/m for the single-phase two-conductor line shown in Figure.



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Transmission Line Parameters

Inductance: Composite Conductors and Unequal Spacing

- **Example 2:**

A single-phase line operating at 60 Hz consists of two 4/0 12-strand copper conductors with 1.5 m spacing between conductor centers. The line length is 32 km. Determine the total inductance in H and the total inductive reactance in W.

The GMR of a 4/0 12-strand copper conductor is $D_{xx}=D_{yy}$=0.01750 ft or 0.5334 cm.

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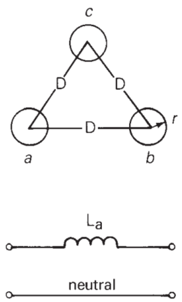
Transmission Line Parameters

Inductance: 3 Phase 3 Wire Line with Equal Phase Spacing

- A three-phase three-wire line consists of three solid cylindrical conductors a, b, c, each with radius *r*, and with equal phase spacing *D* between any two conductors.

- To determine inductance, assume balanced positive-sequence currents I_a, I_b, I_c that satisfy;

$$I_a + I_b + I_c = 0$$



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Transmission Line Parameters

Inductance: 3 Phase 3 Wire Line with Equal Phase Spacing

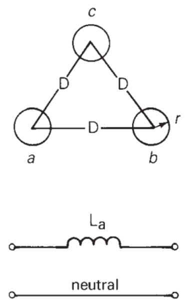
- The total flux linking the phase a conductor is equal to;

$$\lambda_a = 2 \times 10^{-7}\left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D} + I_c \ln \frac{1}{D}\right)$$

- Using $(I_b + I_c) = -I_a$

$$\lambda_a = 2 \times 10^{-7}\left(I_a \ln \frac{1}{r'} - I_a \ln \frac{1}{D}\right)$$

$$= 2 \times 10^{-7} I_a \ln \frac{D}{r'} \quad \text{Wb-t/m}$$



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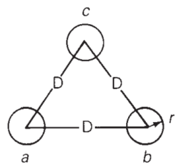
Transmission Line Parameters

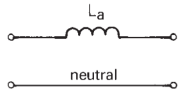
Inductance: 3 Phase 3 Wire Line with Equal Phase Spacing

- The inductance of phase a is;

$$L_a = \frac{\lambda_a}{I_a} = 2 \times 10^{-7} \ln \frac{D}{r'} \quad \text{H/m per phase}$$

- Due to symmetry, the same result is obtained for $L_b = \lambda_b / I_b$ and for $L_c = \lambda_c / I_c$.
- However, only one phase need be considered for balanced three phase operation of this line, since the flux linkages of each phase have equal magnitudes and 120° displacement.





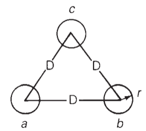
Transmission Line Parameters

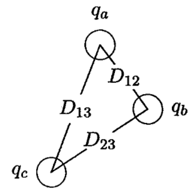
Inductance: Transposition

- To calculate inductance for three-phase lines with stranded conductors and equal phase spacing, r' is replaced by the conductor GMR in;

$$L_a = \frac{\lambda_a}{I_a} = 2 \times 10^{-7} \ln \frac{D}{r'} \quad \text{H/m per phase}$$

- If the spacings between phases are unequal, then balanced positive-sequence flux linkages are not obtained from balanced positive-sequence currents. Instead, unbalanced flux linkages occur, and the phase inductances are unequal.





Transmission Line Parameters

Inductance: Transposition

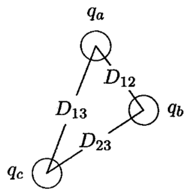
$$\lambda_a = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D_{12}} + I_c \ln \frac{1}{D_{13}} \right)$$

$$\lambda_b = 2 \times 10^{-7} \left(I_a \ln \frac{1}{D_{12}} + I_b \ln \frac{1}{r'} + I_c \ln \frac{1}{D_{23}} \right)$$

$$\lambda_c = 2 \times 10^{-7} \left(I_a \ln \frac{1}{D_{13}} + I_b \ln \frac{1}{D_{23}} + I_c \ln \frac{1}{r'} \right)$$

$$\boldsymbol{\lambda} = \boldsymbol{L}\boldsymbol{I}$$

$$L = 2 \times 10^{-7} \begin{bmatrix} \ln \frac{1}{r'} & \ln \frac{1}{D_{12}} & \ln \frac{1}{D_{13}} \\ \ln \frac{1}{D_{12}} & \ln \frac{1}{r'} & \ln \frac{1}{D_{23}} \\ \ln \frac{1}{D_{13}} & \ln \frac{1}{D_{23}} & \ln \frac{1}{r'} \end{bmatrix}$$



Transmission Line Parameters

Inductance: Transposition

- For balanced three phase system, currents can be represented with taking I_a as reference;

$$I_b = I_a \angle 240° = a^2 I_a$$

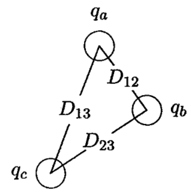
$$I_c = I_a \angle 120° = a I_a$$

where $a = 1\angle 120°$ and $a^2 = 1\angle 240°$

$$L_a = \frac{\lambda_a}{I_a} = 2 \times 10^{-7} \left(\ln \frac{1}{r'} + a^2 \ln \frac{1}{D_{12}} + a \ln \frac{1}{D_{13}} \right)$$

$$L_b = \frac{\lambda_b}{I_b} = 2 \times 10^{-7} \left(a \ln \frac{1}{D_{12}} + \ln \frac{1}{r'} + a^2 \ln \frac{1}{D_{23}} \right)$$

$$L_c = \frac{\lambda_c}{I_c} = 2 \times 10^{-7} \left(a^2 \ln \frac{1}{D_{13}} + a \ln \frac{1}{D_{23}} + \ln \frac{1}{r'} \right)$$



Transmission Line Parameters

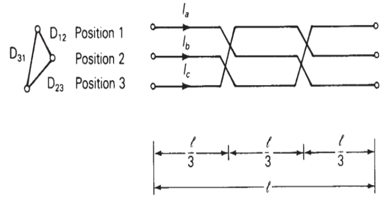
Inductance: Transposition

- If the spacings between phases are unequal, the phase inductances are not equal (unbalanced) and contain an imaginary term due to the mutual inductance.
- In order to balance the inductance of phase conductors, the conductor positions can be exchanged along the line.
- This technique is called as ***transposition***.
- A completely transposed three-phase line is transposed at two locations such that each phase occupies each position for one-third of the line length.

Transmission Line Parameters

Inductance: Transposition

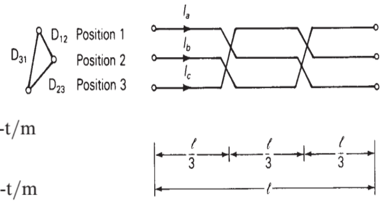
- The line shown in figure is transposed at two locations such that each phase occupies each position for one-third of the line length. Conductor positions are denoted 1, 2, 3 with distances D_{12}, D_{23}, D_{31} between positions.
- The conductors are identical, each with GMR denoted D_S.



Transmission Line Parameters

Inductance: Transposition

- To calculate inductance of this line, assume balanced positive-sequence currents I_a, I_b and I_c for which $I_a+I_b+I_c=0$.
- The total flux linkings of the phase a conductor while it is in position 1, 2 and 3 are;



$$\lambda_{a1} = 2\times 10^{-7}\left[I_a \ln\frac{1}{D_S} + I_b \ln\frac{1}{D_{12}} + I_c \ln\frac{1}{D_{31}}\right] \quad \text{Wb-t/m}$$

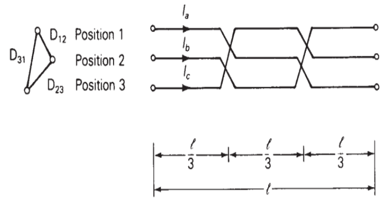
$$\lambda_{a2} = 2\times 10^{-7}\left[I_a \ln\frac{1}{D_S} + I_b \ln\frac{1}{D_{23}} + I_c \ln\frac{1}{D_{12}}\right] \quad \text{Wb-t/m}$$

$$\lambda_{a3} = 2\times 10^{-7}\left[I_a \ln\frac{1}{D_S} + I_b \ln\frac{1}{D_{31}} + I_c \ln\frac{1}{D_{23}}\right] \quad \text{Wb-t/m}$$

Transmission Line Parameters

Inductance: Transposition

- The average of the flux linkages is



$$\lambda_a = \frac{\lambda_{a1}\left(\frac{l}{3}\right) + \lambda_{a2}\left(\frac{l}{3}\right) + \lambda_{a3}\left(\frac{l}{3}\right)}{l} = \frac{\lambda_{a1}+\lambda_{a2}+\lambda_{a3}}{3}$$

$$= \frac{2\times 10^{-7}}{3}\left[3I_a \ln\frac{1}{D_S} + I_b \ln\frac{1}{D_{12}D_{23}D_{31}} + I_c \ln\frac{1}{D_{12}D_{23}D_{31}}\right]$$

- Using $(I_b+I_c)=-I_a$,

$$\lambda_a = \frac{2\times 10^{-7}}{3}\left[3I_a \ln\frac{1}{D_S} - I_a \ln\frac{1}{D_{12}D_{23}D_{31}}\right]$$

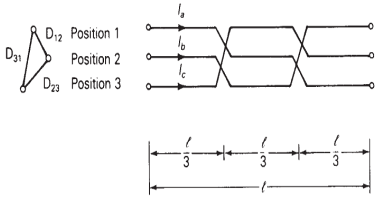
$$= 2\times 10^{-7} I_a \ln\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{D_S} \quad \text{Wb-t/m}$$

Transmission Line Parameters

Inductance: Transposition

- The average inductance of phase a is

$$L_a = \frac{\lambda_a}{I_a} = 2 \times 10^{-7} \ln \frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{D_S} \quad \text{H/m per phase}$$



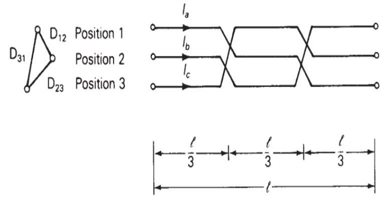
- Same result can be obtained for L_b and L_c. However, only one phase need be considered for balanced three-phase operation of a completely transposed three-phase line.

Transmission Line Parameters

Inductance: Transposition

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_S} \quad \text{H/m}$$

$$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$



- D_{eq}, the cube root of the product of the three-phase spacings, is the geometric mean distance between phases.
- D_S is the conductor GMR for stranded conductors, or *r'* for solid cylindrical conductors.

Transmission Line Parameters

Inductance: Transposition

- **Example 3**

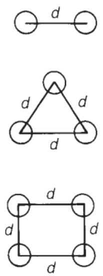
A completely transposed 60-Hz three-phase line has flat horizontal phase spacing with 10 m between adjacent conductors. The conductors are 806 mm^2 (1,590,000 cmil) ACSR with 54/3 stranding. Line length is 200 km. Determine the inductance in H and the inductive reactance in Ω. The GMR of a 806 mm^2 (1,590,000 cmil) 54/3 ACSR conductor is 0.0159 m (0.052 ft).

Transmission Line Parameters

Inductance: Bundled Conductors

- Figure shows common EHV bundles consisting of two, three, or four conductors.
- To calculate inductance in bundled conductors, D_S is replaced by the GMR of the bundle. Since the bundle constitutes a composite conductor, calculation of bundle GMR is given by;

$$D_{xx} = \sqrt[N^2]{\prod_{k=1}^{N} \prod_{m=1}^{N} D_{km}}$$



Transmission Line Parameters

Inductance: Bundled Conductors

- If the conductors are stranded and the bundle spacing d is large compared to the conductor outside radius, each stranded conductor is first replaced by an equivalent solid cylindrical conductor with GMR=D_S. Then the bundle is replaced by one equivalent conductor with GMR=D_{SL} which is calculated for n=2, 3, 4 … as follows:

Two-conductor bundle:

$$D_{SL} = \sqrt[4]{(D_S \times d)^2} = \sqrt{D_S d}$$

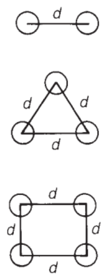
Three-conductor bundle:

$$D_{SL} = \sqrt[9]{(D_S \times d \times d)^3} = \sqrt[3]{D_S d^2}$$

Four-conductor bundle:

$$D_{SL} = \sqrt[16]{(D_S \times d \times d \times d\sqrt{2})^4} = 1.091\sqrt[4]{D_S d^3}$$

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_{SL}} \quad \text{H/m}$$



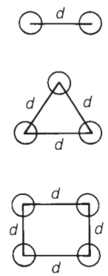
Transmission Line Parameters

Inductance: Bundled Conductors

- The inductance is calculated for bundled conductors as;

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_{SL}} \quad \text{H/m}$$

- If the phase spacings are large compared to the bundle spacing, then sufficient accuracy for D_{eq} is obtained by using the distances between bundle centers.

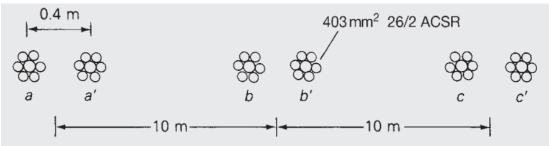


Transmission Line Parameters

Inductance: Bundled Conductors

- **Example 4:**

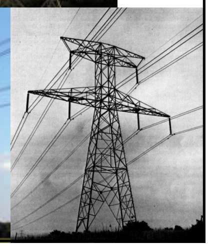
Each of the 806 mm² conductors in Example 3 is replaced by two 403 mm² ACSR 26/2 conductors, as shown in Figure. Bundle spacing is 0.40 m. Flat horizontal spacing is retained, with 10 m between adjacent bundle centers. Calculate the inductive reactance of the line and compare it with that of Example 3.



Transmission Line Parameters

Inductance: Double Circuit Three Phase Lines

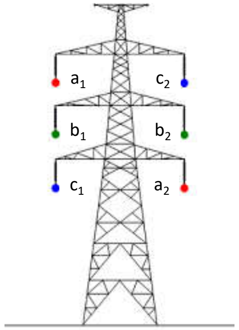
- A three phase double circuit line consists of two identical three phase circuits.
- Double circuits are used where greater **reliability** is needed.
- This method of transmission enables the transfer of more power over a particular distance.



Transmission Line Parameters

Inductance: Double Circuit Three Phase Lines

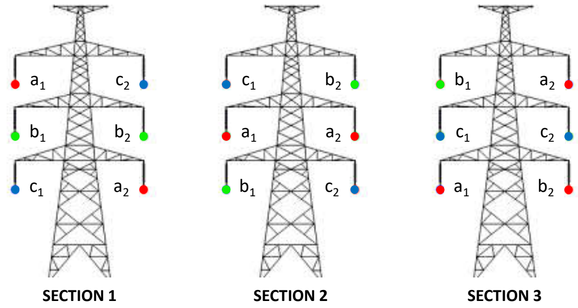
- The circuits are operated with a_1-a_2, b_1-b_2 and c_1-c_2 in parallel.
- Because of the geometrical differences between conductors, voltage drop due to line inductance will be unbalanced.
- To achieve balance operation, each phase conductor must be transposed within its group and with respect to the parallel three phase line.



Transmission Line Parameters

Inductance: Double Circuit Three Phase Lines

The arrangement of conductors of a transposed double circuit line:



Transmission Line Parameters

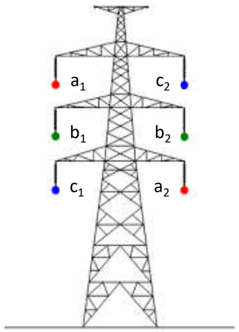
Inductance: Double Circuit Three Phase Lines

- By grouping the identical phases together, GMD between each phase group can be found as;

$$D_{AB} = \sqrt[4]{D_{a_1b_1}D_{a_1b_2}D_{a_2b_1}D_{a_2b_2}}$$
$$D_{BC} = \sqrt[4]{D_{b_1c_1}D_{b_1c_2}D_{b_2c_1}D_{b_2c_2}}$$
$$D_{AC} = \sqrt[4]{D_{a_1c_1}D_{a_1c_2}D_{a_2c_1}D_{a_2c_2}}$$

- The equivalent GMD per phase is equal to;

$$GMD = \sqrt[3]{D_{AB}D_{BC}D_{AC}}$$



Transmission Line Parameters

Inductance: Double Circuit Three Phase Lines

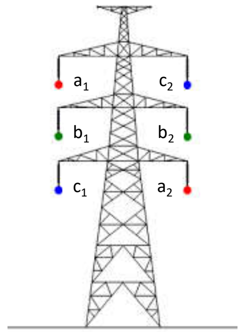
- The GMR of each phase group is

$$D_{SA} = \sqrt[4]{(D_s^b D_{a_1a_2})^2} = \sqrt{D_s^b D_{a_1a_2}}$$
$$D_{SB} = \sqrt[4]{(D_s^b D_{b_1b_2})^2} = \sqrt{D_s^b D_{b_1b_2}}$$
$$D_{SC} = \sqrt[4]{(D_s^b D_{c_1c_2})^2} = \sqrt{D_s^b D_{c_1c_2}}$$

where D_s^b is the geometric mean radius of the bundled conductors.

- The equivalent GMR for calculating the per-phase inductance is equal to;

$$GMR_L = \sqrt[3]{D_{SA}D_{SB}D_{SC}}$$

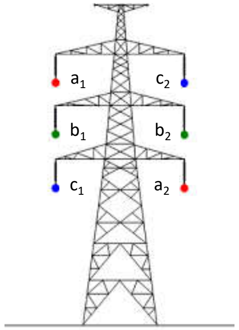


Transmission Line Parameters

Inductance: Double Circuit Three Phase Lines

- The inductance per phase in millihenries per kilometer is equal to;

$$L = 0.2 \ln \frac{GMD}{GMR_L} \quad \text{mH/km}$$



Transmission Line Parameters

Capacitance

- The capacitance between conductors in a medium with constant permittivity ε can be obtained by determining the following:

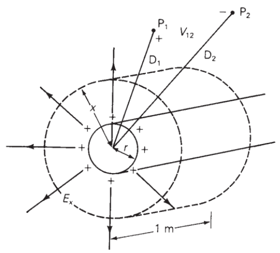
 1. Electric field strength E, from Gauss's law
 2. Voltage between conductors
 3. Capacitance from charge per unit volt (C = q/V)

Transmission Line Parameters

Capacitance: Solid Cylindrical Conductor

- Figure shows a solid cylindrical conductor with radius *r* and with positive charge *q* coloumbs per meter uniformly distributed on the conductor surface.

- For simplicity, assume that the conductor is,
  - sufficiently long that end effects are negligible.
  - a perfect conductor (that is, zero resistivity, *ρ*=0).

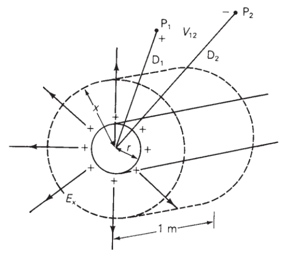


Transmission Line Parameters

Capacitance: Solid Cylindrical Conductor

Internal Electric Field

- Inside the perfect conductor, Ohm's law gives $E_{int}= \rho J=0$. That is, the internal electric field E_{int} is zero.



Transmission Line Parameters

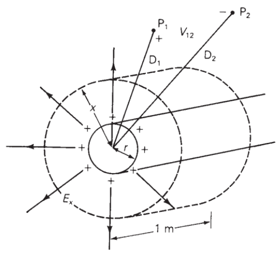
Capacitance: Solid Cylindrical Conductor

External Electric Field

- To determine the electric field outside the conductor, select the cylinder with radius r and with 1-meter length.
- Due to the uniform charge distribution, the electric field strength E_x is constant on the cylinder. Also, there is no tangential component of E_x, so the electric field is radial to the conductor. Then the electric field strength E_x is equal to;

$$E_x = \frac{q}{2\pi\varepsilon x} \quad \text{V/m}$$

where $\varepsilon = \varepsilon_0 = 8.854 \times 10^{-12}$ F/m for a conductor in free space.



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Transmission Line Parameters

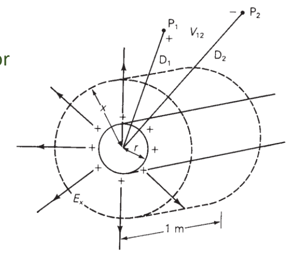
Capacitance: Solid Cylindrical Conductor

External Electric Field

- Concentric cylinders surrounding the conductor are constant potential surfaces. The potential difference between two concentric cylinders at distances D_1 and D_2 from the conductor center is,

$$V_{12} = \int_{D_1}^{D_2} E_x \, dx$$

$$V_{12} = \int_{D_1}^{D_2} \frac{q}{2\pi\varepsilon x} dx = \frac{q}{2\pi\varepsilon} \ln \frac{D_2}{D_1} \quad \text{volts}$$



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Transmission Line Parameters

Capacitance: Solid Cylindrical Conductor

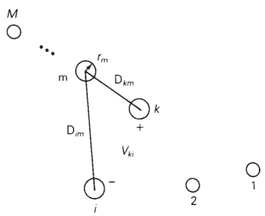
The Array of Solid Cylindrical Conductors

- Assume that each conductor m has an ac charge q_m C/m uniformly distributed along the conductor. The voltage V_{kim} between conductors k and i due to the charge q_m acting alone is

$$V_{kim} = \frac{q_m}{2\pi\varepsilon} \ln \frac{D_{im}}{D_{km}} \quad \text{volts}$$

where $D_{mm} = r_m$ when k = m or $i = m$.

- The distortion of the electric field in the vicinity of the other conductors is neglected.



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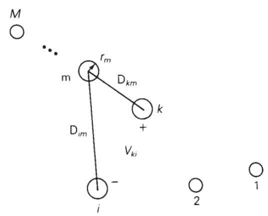
Transmission Line Parameters

Capacitance: Solid Cylindrical Conductor

The Array of Solid Cylindrical Conductors

- Using superposition, the voltage V_{ki} between conductors k and i due to all the charges is

$$V_{ki} = \frac{1}{2\pi\varepsilon} \sum_{m=1}^{M} q_m \ln \frac{D_{im}}{D_{km}} \quad \text{volts}$$



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Transmission Line Parameters

Capacitance: Single Phase Two Wire Line

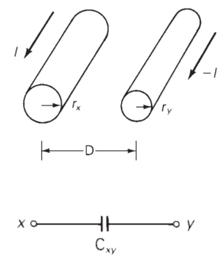
- Assume that the conductors are energized by a voltage source such that conductor x has a uniform charge *q* C/m and, assuming conservation of charge, conductor y has an equal quantity of negative charge -q.
- Potential difference between conductor x and conductor y is;

$$V_{xy} = \frac{1}{2\pi\varepsilon}\left[q \ln \frac{D_{yx}}{D_{xx}} - q \ln \frac{D_{yy}}{D_{xy}}\right]$$

$$= \frac{q}{2\pi\varepsilon} \ln \frac{D_{yx}D_{xy}}{D_{xx}D_{yy}}$$

- Using $D_{xy}=D_{yx}=D$, $D_{xx}=r_x$, and $D_{yy}=r_y$,

$$V_{xy} = \frac{q}{\pi\varepsilon} \ln \frac{D}{\sqrt{r_x r_y}} \quad \text{volts}$$



Transmission Line Parameters

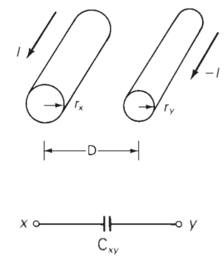
Capacitance: Single Phase Two Wire Line

- For a 1-meter line length, the capacitance between conductors is

$$C_{xy} = \frac{q}{V_{xy}} = \frac{\pi\varepsilon}{\ln\left(\frac{D}{\sqrt{r_x r_y}}\right)} \quad \text{F/m line-to-line}$$

- If $r_x=r_y=r$,

$$C_{xy} = \frac{\pi\varepsilon}{\ln(D/r)} \quad \text{F/m line-to-line}$$



Transmission Line Parameters

Capacitance: Single Phase Two Wire Line

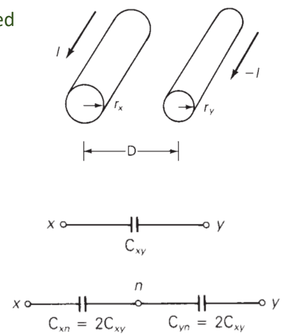
- If the two-wire line is supplied by a transformer with a grounded center tap, then the voltage between each conductor and ground is one-half

$$V_{xn} = V_{yn} = \frac{V_{xy}}{2}$$

- The capacitance from either line to the grounded neutral is

$$C_n = C_{xn} = C_{yn} = \frac{q}{V_{xn}} = 2C_{xy}$$

$$= \frac{2\pi\varepsilon}{\ln(D/r)} \quad \text{F/m line-to-neutral}$$



Transmission Line Parameters

Capacitance: Composite Conductors and Unequal Spacing

- Practical conductors have a small internal electric field because of their resistivities. As a result, the external electric field is slightly altered near the conductor surfaces.

- In addition, the electric field near the surface of a stranded conductor is not the same as that of a solid cylindrical conductor. However , it is normal practice when calculating line capacitance to replace a stranded conductor by a perfectly conducting solid cylindrical conductor whose radius equals the outside radius of the stranded conductor. The resulting error in capacitance is small since only the electric field near the conductor surfaces is affected.

Transmission Line Parameters

Capacitance: 3 Phase 3 Wire Line with Equal Phase Spacing

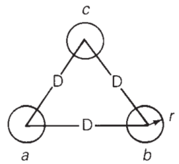
- In order to determine the positive-sequence capacitance, the effect of earth and neutral conductors is neglected and the positive-sequence charges q_a, q_b, q_c are assumed such that $q_a+q_b+q_c=0$. the voltage V_{ab} between conductors a and b is

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln \frac{D_{ba}}{D_{aa}} + q_b \ln \frac{D_{bb}}{D_{ab}} + q_c \ln \frac{D_{bc}}{D_{ac}}\right]$$

- Using $D_{aa}=D_{bb}=r$, and $D_{ab}=D_{ba}=D_{ca}=D_{cb}=D$, the voltage V_{ab} becomes

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln \frac{D}{r} + q_b \ln \frac{r}{D} + q_c \ln \frac{D}{D}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln \frac{D}{r} + q_b \ln \frac{r}{D}\right] \quad \text{volts}$$



- *Note that the third term in the equation of V_{ab} is zero because conductors a and b are equidistant from conductor c. Thus, conductors a and b lie on a constant potential cylinder for the electric field due to q_c.*

Transmission Line Parameters

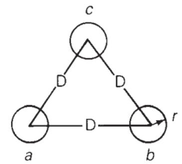
Capacitance: 3 Phase 3 Wire Line with Equal Phase Spacing

- Similarly, the voltage V_{ac} between conductors a and c is

$$V_{ac} = \frac{1}{2\pi\varepsilon}\left[q_a \ln \frac{D_{ca}}{D_{aa}} + q_b \ln \frac{D_{cb}}{D_{ab}} + q_c \ln \frac{D_{cc}}{D_{ac}}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln \frac{D}{r} + q_b \ln \frac{D}{D} + q_c \ln \frac{r}{D}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln \frac{D}{r} + q_c \ln \frac{r}{D}\right] \quad \text{volts}$$



Transmission Line Parameters

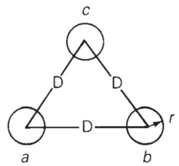
Capacitance: 3 Phase 3 Wire Line with Equal Phase Spacing

- Recall that for balanced positive-sequence voltages,

$$V_{ab} = \sqrt{3}V_{an}\underline{/+30^\circ} = \sqrt{3}V_{an}\left[\frac{\sqrt{3}}{2} + j\frac{1}{2}\right]$$

$$V_{ac} = -V_{ca} = \sqrt{3}V_{an}\underline{/-30^\circ} = \sqrt{3}V_{an}\left[\frac{\sqrt{3}}{2} - j\frac{1}{2}\right]$$

$$V_{ab} + V_{ac} = 3V_{an}$$



- With using balanced positive-sequence voltage equations, the voltage V_{an} between conductors a and neutral becomes;

$$V_{an} = \frac{1}{3}\left(\frac{1}{2\pi\varepsilon}\right)\left[2q_a \ln \frac{D}{r} + (q_b + q_c) \ln \frac{r}{D}\right]$$

Transmission Line Parameters

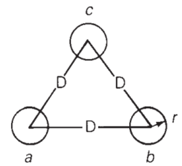
Capacitance: 3 Phase 3 Wire Line with Equal Phase Spacing

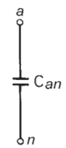
- Using $q_b+q_c=-q_a$ for the three phase balance situation, V_{an} becomes;

$$V_{an} = \frac{1}{2\pi\varepsilon} q_a \ln \frac{D}{r} \quad \text{volts}$$

- The capacitance-to-neutral per line length is

$$C_{an} = \frac{q_a}{V_{an}} = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{r}\right)} \quad \text{F/m line-to-neutral}$$





- Due to symmetry, the same result is obtained for $C_{bn}=q_b/V_{bn}$ and $C_{cn}=q_c/V_{cn}$. For balanced three-phase operation, however, only one phase need be considered.

Transmission Line Parameters

Capacitance: Unequal Phase Spacing

- For three-phase lines with unequal phase spacing, balanced positive sequence voltages are not obtained with balanced positive-sequence charges.
- Instead, unbalanced line-to-neutral voltages occur, and the phase- to-neutral capacitances are unequal. Balance can be restored by transposing the line such that each phase occupies each position for one-third of the line length.

Transmission Line Parameters

Capacitance: Transposition

- If the equations of V_{ab} and V_{ac} are written for each position in the transposition cycle, and are then averaged, the resulting capacitance becomes;

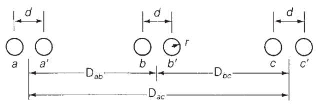
$$C_{an} = \frac{2\pi\varepsilon}{\ln(D_{eq}/r)} \quad \text{F/m}$$

$$D_{eq} = \sqrt[3]{D_{ab} D_{bc} D_{ac}}$$

Transmission Line Parameters

Capacitance: Bundled Conductors

- Figure shows a bundled conductor line with two conductors per bundle.
- To determine the capacitance of this line, assume balanced positive sequence charges q_a, q_b, q_c for each phase such that $q_a+q_b+q_c=0$.
- Assume that the conductors in each bundle, which are in parallel, share the charges equally. Thus conductors a and a' each have the charge $q_a/2$.
- Also assume that the phase spacings are much larger than the bundle spacings so that D_{ab} may be used instead of $(D_{ab}-d)$ or $(D_{ab}+d)$.



Transmission Line Parameters

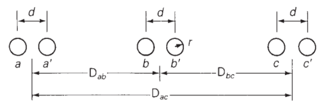
Capacitance: Bundled Conductors

- The voltage between conductors a and b, V_{ab} is equal to;

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left[\frac{q_a}{2}\ln\frac{D_{ba}}{D_{aa}} + \frac{q_a}{2}\ln\frac{D_{ba'}}{D_{aa'}} + \frac{q_b}{2}\ln\frac{D_{bb}}{D_{ab}} + \frac{q_b}{2}\ln\frac{D_{bb'}}{D_{ab'}} + \frac{q_c}{2}\ln\frac{D_{bc}}{D_{ac}} + \frac{q_c}{2}\ln\frac{D_{bc'}}{D_{ac'}}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[\frac{q_a}{2}\left(\ln\frac{D_{ab}}{r} + \ln\frac{D_{ab}}{d}\right) + \frac{q_b}{2}\left(\ln\frac{r}{D_{ab}} + \ln\frac{d}{D_{ab}}\right) + \frac{q_c}{2}\left(\ln\frac{D_{bc}}{D_{ac}} + \ln\frac{D_{bc}}{D_{ac}}\right)\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a\ln\frac{D_{ab}}{\sqrt{rd}} + q_b\ln\frac{\sqrt{rd}}{D_{ab}} + q_c\ln\frac{D_{bc}}{D_{ac}}\right]$$

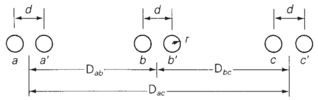


Transmission Line Parameters
Capacitance: Bundled Conductors

- For a transposed line, derivation of the capacitance would yield;

$$C_{an} = \frac{2\pi\varepsilon}{\ln(D_{eq}/D_{SC})} \quad \text{F/m}$$

$$D_{SC} = \sqrt{rd}$$



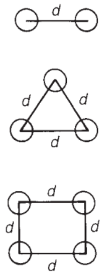
Transmission Line Parameters
Capacitance: Bundled Conductors

- The capacitance equation is analogous to the inductance equation.
- In both inductance and capacitance equations, D_{eq} is the geometric mean of the distances between phases.
- Also, D_{SC} in the capacitance equations is analogous to D_{SL} in the inductance equations, except that the conductor outside radius r replaces the conductor GMR D_S.

$$D_{SC} = \sqrt{rd} \quad \text{for a two-conductor bundle}$$

$$D_{SC} = \sqrt[3]{rd^2} \quad \text{for a three-conductor bundle}$$

$$D_{SC} = 1.091\sqrt[4]{rd^3} \quad \text{for a four-conductor bundle}$$



Transmission Line Parameters
Capacitance: Bundled Conductors

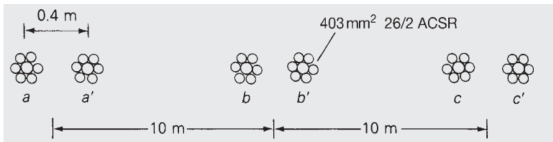
- **Example 5**

A single-phase line operating at 60 Hz consists of two 4/0 12-strand copper conductors with 1.5 m spacing between conductor centers. The line length is 32km. Determine the line-to-line capacitance in F and the line-to-line admittance in S. If the line voltage is 20 kV, determine the reactive power in kVAr supplied by this capacitance. The radius of a 4/0 12-strand copper conductor is 0.7 cm.

Transmission Line Parameters
Capacitance: Bundled Conductors

- **Example 6**

The three-phase line consists of two 403 mm^2 ACSR 26/2 conductors as shown in Figure. The flat horizontal spacing is retained, with 10 m between adjacent bundle centers and the bundle spacing is 0.40 m. Determine the capacitance-to-neutral in F and the shunt admittance-to-neutral in S. If the line voltage is 345 kV, determine the charging current in kA per phase and the total reactive power in MVAr supplied by the line capacitance . Assume balanced positive sequence voltages. the raidus of a 403 mm^2 26/2 ACSR conductor is 0.0141 m.



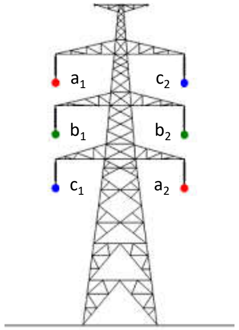
Transmission Line Parameters
Capacitance: Double Circuit Three Phase Lines

- The expression of GMD is the same as found for inductance calculation.
- By grouping the identical phases together, GMD between each phase group can be found as;

$$D_{AB} = \sqrt[4]{D_{a_1b_1}D_{a_1b_2}D_{a_2b_1}D_{a_2b_2}}$$
$$D_{BC} = \sqrt[4]{D_{b_1c_1}D_{b_1c_2}D_{b_2c_1}D_{b_2c_2}}$$
$$D_{AC} = \sqrt[4]{D_{a_1c_1}D_{a_1c_2}D_{a_2c_1}D_{a_2c_2}}$$

- The equivalent GMD per phase is equal to;

$$GMD = \sqrt[3]{D_{AB}D_{BC}D_{AC}}$$



Transmission Line Parameters
Capacitance: Double Circuit Three Phase Lines

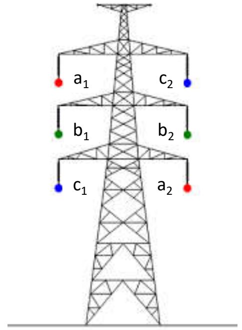
- The GMR_C of each phase group is

$$D_{SAc} = \sqrt{D_{Sc}^{b}D_{a1a2}}$$
$$D_{SBc} = \sqrt{D_{Sc}^{b}D_{b1b2}}$$
$$D_{SBc} = \sqrt{D_{Sc}^{b}D_{c1c2}}$$

where D_{Sc}^{b} is the geometric mean radius of the bundled conductors for capacitance calculations.

- The equivalent GMR for calculating the per-phase capacitance, D_{Sc} is equal to;

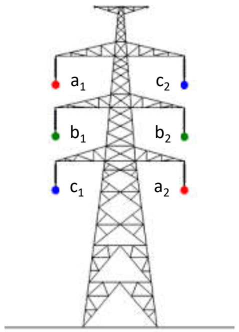
$$D_{Sc} = \sqrt[3]{D_{SAc}D_{SBc}D_{SCc}}$$



Transmission Line Parameters
Capacitance: Double Circuit Three Phase Lines

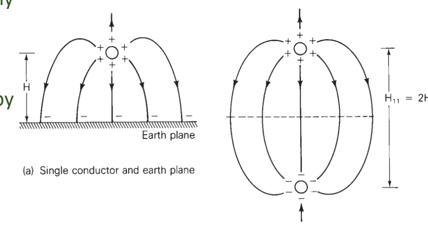
- The capacitance per phase in Farad per kilometer is equal to;

$$C_{an} = \frac{2\pi\varepsilon_0}{\ln\frac{GMD}{D_{Sc}}} \quad F/m$$



Transmission Line Parameters
Capacitance: Earth Effect

- We approximate the earth surface as a perfectly conducting horizontal plane, even though the earth under the line may have irregular terrain and resistivities.
- The effect of the earth plane is accounted for by the method of images.



(a) Single conductor and earth plane

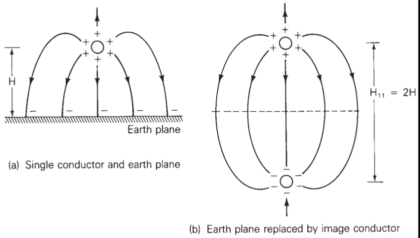
(b) Earth plane replaced by image conductor

Transmission Line Parameters

Capacitance: Earth Effect

The method of images:

- Consider a single conductor with uniform charge distribution and with height H above a perfectly conducting earth plane, as shown in Figure.
- When the conductor has a positive charge, an equal quantity of negative charge is induced on the earth. The electric field lines will originate from the positive charges on the conductor and terminate at the negative charges on the earth. Also, the electric field lines are perpendicular to the surfaces of the conductor and earth.



(a) Single conductor and earth plane

(b) Earth plane replaced by image conductor

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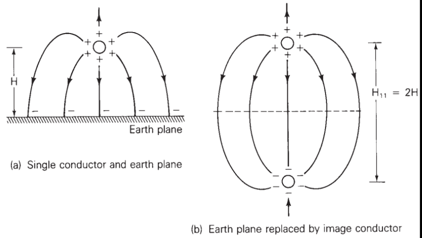
Transmission Line Parameters

Capacitance: Earth Effect

The method of images:

- Now replace the earth by the image conductor shown in Figure(b), which has the same radius as the original conductor, lies directly below the original conductor with conductor separation $H_{11}=2H$, and has an equal quantity of negative charge.
- The electric field above the dashed line representing the location of the removed earth plane in Figure(b) is identical to the electric field above the earth plane in Figure(a). Therefore, the voltage between any two points above the earth is the same in both figures.



(a) Single conductor and earth plane

(b) Earth plane replaced by image conductor

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Transmission Line Parameters

Capacitance: Earth Effect

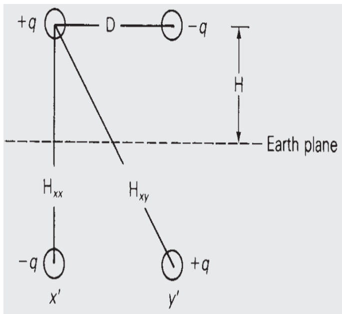
Single Phase Transmission Line

- The earth plane is replaced by a separate image conductor for each overhead conductor of single phase transmission line as shown in figure.
- The voltage between conductors a and b is

$$V_{xy}=\frac{q}{2\pi\varepsilon}\left[\ln\frac{D_{yx}}{D_{xx}}-\ln\frac{D_{yy}}{D_{xy}}-\ln\frac{H_{yx}}{H_{xx}}+\ln\frac{H_{yy}}{H_{xy}}\right]$$

$$=\frac{q}{2\pi\varepsilon}\left[\ln\frac{D_{yx}D_{xy}}{D_{xx}D_{yy}}-\ln\frac{H_{yx}H_{xy}}{H_{xx}H_{yy}}\right]$$

$$=\frac{q}{\pi\varepsilon}\left[\ln\frac{D}{r}-\ln\frac{H_{xy}}{H_{xx}}\right]$$



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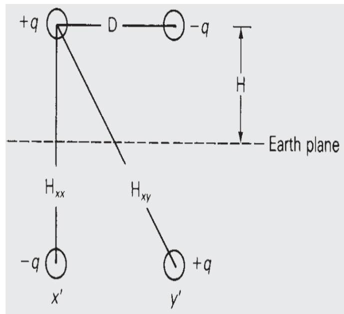
Transmission Line Parameters

Capacitance: Earth Effect

Single Phase Transmission Line

- The line to line capacitance is calculted as;

$$C_{xy}=\frac{q}{V_{xy}}=\frac{\pi\varepsilon}{\ln\frac{D}{r}-\ln\frac{H_{xy}}{H_{xx}}}\quad \mathrm{F/m}$$



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Transmission Line Parameters

Capacitance: Earth Effect

Three Phase Transmission Line

- The earth plane is replaced by a separate image conductor for each overhead conductor of three phase transmission line as shown in figure.
- The line to neutral capacitance of three phase transposed line including earth effect is given by,

$$C_{an} = \frac{2\pi\varepsilon_0}{\ln\frac{D_{eq}}{r} - \ln\frac{H_m}{H_s}} \text{ F/m Line-to-neutral}$$

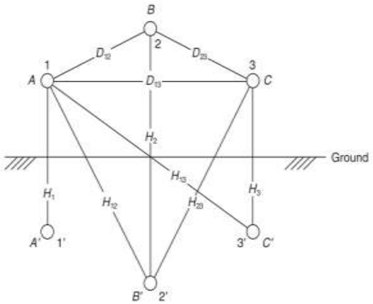
where

$$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{13}} = \text{GMD}$$

r = conductor's outside radius

$$H_m = (H_{12}H_{23}H_{13})^{1/3}$$

$$H_s = (H_1H_2H_3)^{1/3}$$

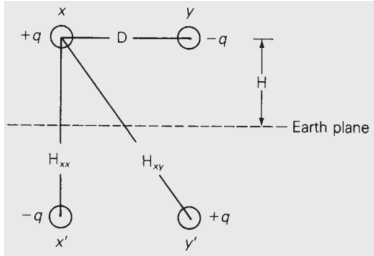


Transmission Line Parameters

Capacitance: Earth Effect

- **Example 7**

A single-phase line operating at 60 Hz consists of two 4/0 12-strand copper conductors with 1.5 m horizontal spacing between conductor centers. The line has 5.49 m average line height. Determine the effect of the earth on capacitance. Assume a perfectly conducting earth plane. The GMR of a 4/0 12-strand copper conductor is 0.5334 cm.



Transmission Line Parameters

Capacitance: Earth Effect

- **Example 8**

5.6 A three-phase 60-Hz line has flat horizontal spacing. The conductors have an outside diameter of 3.28 cm with 12 m between conductors. Determine the capacitive reactance to neutral in ohm-meters and the capacitive reactance of the line in ohms if its length is 125 mi.

Solve Prob. 5.6 while taking account of the effect of ground. Assume that the conductors are horizontally placed 20 m above ground.

Transmission Line Parameters

Review Problems

- **Problem 1**

A stranded conductor consists of seven identical strands each having a radius r as shown in Figure 4.11. Determine the GMR of the conductor in terms of r.

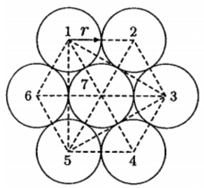


FIGURE 4.11
Cross section of a stranded conductor.

Transmission Line Parameters
Review Problems

- **Problem 2**

A 60-Hz three-phase line composed of one ACSR *Bluejay* conductor per phase has flat horizontal spacing of 11 m between adjacent conductors. Compare the inductive reactance in ohms per kilometer per phase of this line with that of a line using a two-conductor bundle of ACSR 26/7 conductors having the same total cross-sectional area of aluminum as the single-conductor line and 11 m spacing measured from the center of the bundles. The spacing between conductors in the bundle is 40 cm.

Transmission Line Parameters
Review Problems

- **Problem 3**

A 60-Hz three-phase line composed of one ACSR *Bluejay* conductor per phase has flat horizontal spaicng of 11 m between adjacent conductors. Compare the capacitive reactance in ohm-kilometers per phase of this line with that of a line using a two-conductor bundle of ACSR 26/7 conductors having the same total cross-sectional area of aluminum as the single-conductor line and the 11 m spacing measured between bundles. The spacing between conductors in the bundle is 40 cm.

Transmission Line Parameters
Review Problems

- **Problem 3**

A 3 phase, 380kV, 50Hz, 100km long completely transposed line is built with three bundle ACSR Pheasant conductor. ACSR Pheasant conductor has an outside diameter of 35.1 mm and GMR of 13.66 mm. the conductors in bundles are equally spaced with 50 cm. A horizontal tower configuration is used with a phase spacing of 12 m. The height of conductors is 40 m from the earth level.

  - Determine the per phase inductance of the transmission line for three phase balanced conditions.
  - Determine the per phase capacitance of the transmission line for three phase balanced conditions by neglecting the earth effect.
  - Determine the per phase capacitance of the transmission line for three phase balanced conditions by taking account of earth effect. Assume that the earth surface is perfectly conducting horizontal plane.

Transmission Line Parameters
Review Problems

- **Problem 5**

A 345-kV double-circuit three-phase transposed line is composed of two $ACSR$, 1,431,000-cmil, 45/7 Bobolink conductors per phase with vertical conductor configuration as shown in Figure 4.22. The conductors have a diameter of 1.427 in and a GMR of 0.564 in. The bundle spacing in 18 in. Find the inductance and capacitance per phase per kilometer of the line. The following commands

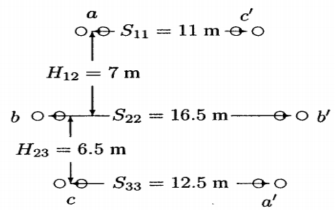


FIGURE 4.22
Conductor layout for Example 4.5.