

**Example 1:** The ventilating fan of the bathroom of a building has a volume flow rate of 30 L/s and runs continuously. If the density of air inside is 1.20 kg/m<sup>3</sup>, determine the mass of air vented out in one day.

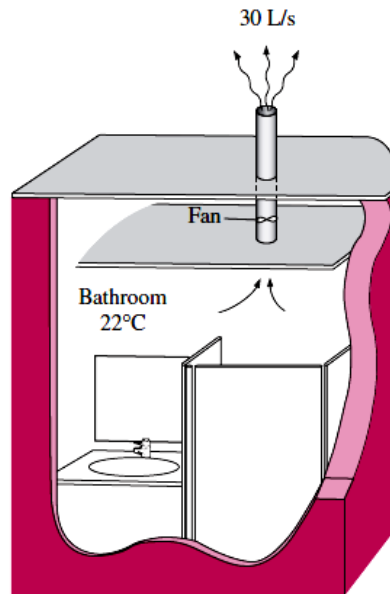


Figure 4-4.

**Assumptions** Flow through the fan is steady.

**Properties** The density of air in the building is given to be 1.20 kg/m<sup>3</sup>.

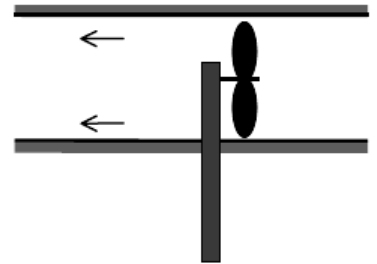
**Analysis** The mass flow rate of air vented out is

$$\dot{m}_{\text{air}} = \rho \dot{V}_{\text{air}} = (1.20 \text{ kg/m}^3)(0.030 \text{ m}^3/\text{s}) = 0.036 \text{ kg/s}$$

Then the mass of air vented out in 24 h becomes

$$m = \dot{m}_{\text{air}} \Delta t = (0.036 \text{ kg/s})(24 \times 3600 \text{ s}) = \mathbf{3110 \text{ kg}}$$

**Discussion** Note that more than 3 tons of air is vented out by a bathroom fan in one day.



**Example 2:** A desktop computer is to be cooled by a fan whose flow rate is 0.34 m<sup>3</sup>/min. Determine the mass flow rate of air through the fan. (The air density is 0.7 kg/m<sup>3</sup>) Also, if the average velocity of air is not to exceed 110 m/min, determine the diameter of the casing of the fan.

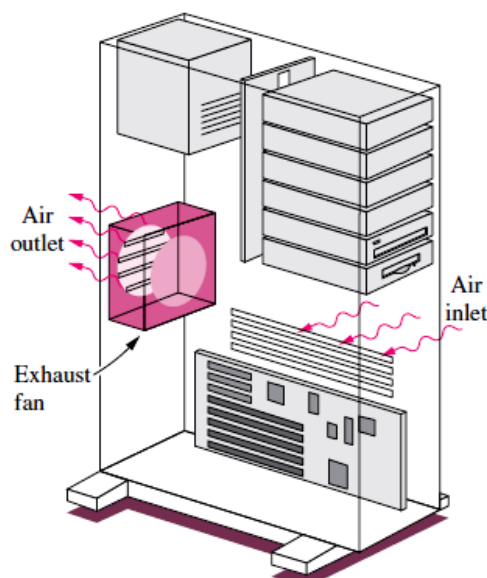


Figure 4-5.

**Assumptions** Flow through the fan is steady.

**Properties** The density of air at a high elevation is given to be  $0.7 \text{ kg/m}^3$ .

**Analysis** The mass flow rate of air is

$$\dot{m}_{\text{air}} = \rho \dot{V}_{\text{air}} = (0.7 \text{ kg/m}^3)(0.34 \text{ m}^3/\text{min}) = 0.238 \text{ kg/min} = \mathbf{0.0040 \text{ kg/s}}$$

If the mean velocity is  $110 \text{ m/min}$ , the diameter of the casing is

$$\dot{V} = AV = \frac{\pi D^2}{4} V \rightarrow D = \sqrt{\frac{4\dot{V}}{\pi V}} = \sqrt{\frac{4(0.34 \text{ m}^3/\text{min})}{\pi(110 \text{ m/min})}} = \mathbf{0.063 \text{ m}}$$

Therefore, the diameter of the casing must be at least  $6.3 \text{ cm}$  to ensure that the mean velocity does not exceed  $110 \text{ m/min}$ .

**Example 3:** Consider a river flowing toward a lake at an average velocity of  $3 \text{ m/s}$  at a rate of  $500 \text{ m}^3/\text{s}$  at a location  $90 \text{ m}$  above the lake surface. Determine the total mechanical energy of the river water per unit mass and the power generation potential of the entire river at that location.

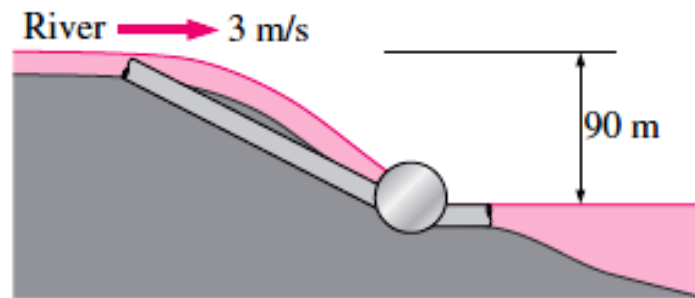


Figure 4-7.

**Assumptions** 1 The elevation given is the elevation of the free surface of the river. 2 The velocity given is the average velocity. 3 The mechanical energy of water at the turbine exit is negligible.

**Properties** We take the density of water to be  $\rho = 1000 \text{ kg/m}^3$ .

**Analysis** Noting that the sum of the flow energy and the potential energy is constant for a given fluid body, we can take the elevation of the entire river water to be the elevation of the free surface, and ignore the flow energy. Then the total mechanical energy of the river water per unit mass becomes

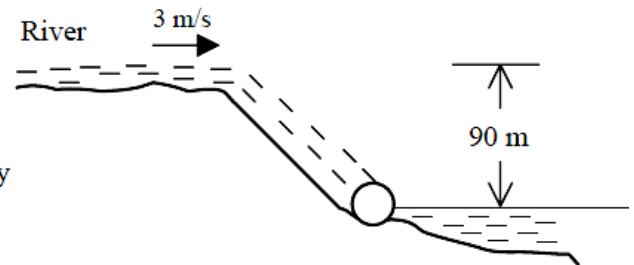
$$\begin{aligned} e_{\text{mech}} &= pe + ke = gh + \frac{V^2}{2} \\ &= \left( (9.81 \text{ m/s}^2)(90 \text{ m}) + \frac{(3 \text{ m/s})^2}{2} \right) \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) \\ &= \mathbf{0.887 \text{ kJ/kg}} \end{aligned}$$

The power generation potential of the river water is obtained by multiplying the total mechanical energy by the mass flow rate,

$$\dot{m} = \rho \dot{V} = (1000 \text{ kg/m}^3)(500 \text{ m}^3/\text{s}) = 500,000 \text{ kg/s}$$

$$\dot{W}_{\text{max}} = \dot{E}_{\text{mech}} = \dot{m}e_{\text{mech}} = (500,000 \text{ kg/s})(0.887 \text{ kJ/kg}) = 444,000 \text{ kW} = \mathbf{444 \text{ MW}}$$

Therefore,  $444 \text{ MW}$  of power can be generated from this river as it discharges into the lake if its power potential can be recovered completely.



**Example 4:** Water is pumped from a lake to a storage tank 20 m above at a rate of 70 L/s while consuming 20.4 kW of electric power. Disregarding any frictional losses in the pipes and any changes in kinetic energy, determine (a) the overall efficiency of the pump–motor unit and (b) the pressure difference between the inlet and the exit of the pump.

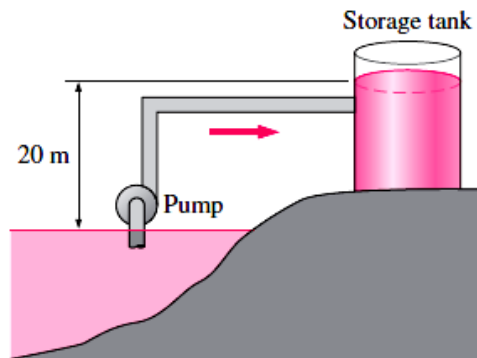


Figure 4-8.

**Assumptions** 1 The elevations of the tank and the lake remain constant. 2 Frictional losses in the pipes are negligible. 3 The changes in kinetic energy are negligible. 4 The elevation difference across the pump is negligible.

**Properties** We take the density of water to be  $\rho = 1000 \text{ kg/m}^3$ .

**Analysis** (a) We take the free surface of the lake to be point 1 and the free surfaces of the storage tank to be point 2. We also take the lake surface as the reference level ( $z_1 = 0$ ), and thus the potential energy at points 1 and 2 are  $pe_1 = 0$  and  $pe_2 = gz_2$ . The flow energy at both points is zero since both 1 and 2 are open to the atmosphere ( $P_1 = P_2 = P_{\text{atm}}$ ). Further, the kinetic energy at both points is zero ( $ke_1 = ke_2 = 0$ ) since the water at both locations is essentially stationary. The mass flow rate of water and its potential energy at point 2 are

$$\dot{m} = \rho \dot{V} = (1000 \text{ kg/m}^3)(0.070 \text{ m}^3/\text{s}) = 70 \text{ kg/s}$$

$$pe_2 = gz_2 = (9.81 \text{ m/s}^2)(20 \text{ m}) \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 0.196 \text{ kJ/kg}$$

Then the rate of increase of the mechanical energy of water becomes

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{m}(e_{\text{mech, out}} - e_{\text{mech, in}}) = \dot{m}(pe_2 - 0) = \dot{m}pe_2 = (70 \text{ kg/s})(0.196 \text{ kJ/kg}) = 13.7 \text{ kW}$$

The overall efficiency of the combined pump-motor unit is determined from its definition,

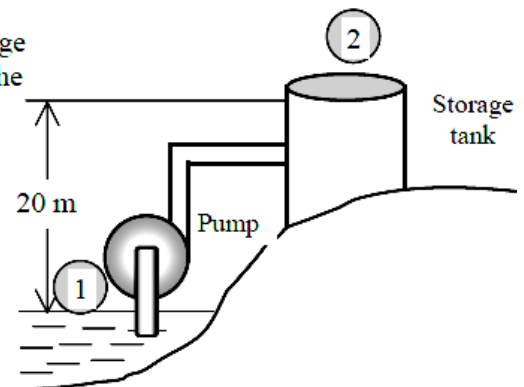
$$\eta_{\text{pump-motor}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{elect, in}}} = \frac{13.7 \text{ kW}}{20.4 \text{ kW}} = 0.672 \quad \text{or} \quad \mathbf{67.2\%}$$

(b) Now we consider the pump. The change in the mechanical energy of water as it flows through the pump consists of the change in the flow energy only since the elevation difference across the pump and the change in the kinetic energy are negligible. Also, this change must be equal to the useful mechanical energy supplied by the pump, which is 13.7 kW:

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{m}(e_{\text{mech, out}} - e_{\text{mech, in}}) = \dot{m} \frac{P_2 - P_1}{\rho} = \dot{V} \Delta P$$

Solving for  $\Delta P$  and substituting,

$$\Delta P = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{V}} = \frac{13.7 \text{ kJ/s}}{0.070 \text{ m}^3/\text{s}} \left( \frac{1 \text{ kPa} \cdot \text{m}^3}{1 \text{ kJ}} \right) = \mathbf{196 \text{ kPa}}$$



Therefore, the pump must boost the pressure of water by 196 kPa in order to raise its elevation by 20 m.

**Example 5:** Water is flowing from a hose attached to a water main at 400 kPa gage. A child places his thumb to cover most of the hose outlet, causing a thin jet of high-speed water to emerge. If the hose is held upward, what is the maximum height that the jet could achieve?

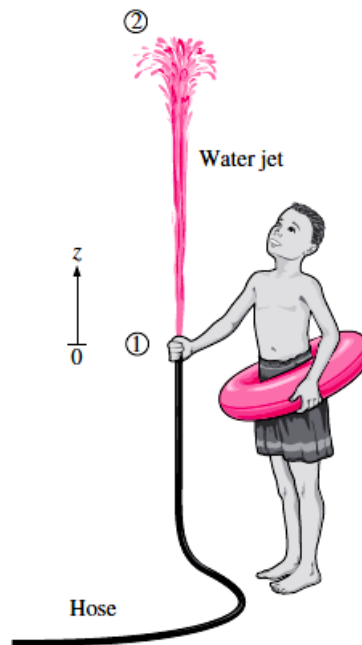


Figure 4-12.

**Solution:**

Assumptions: 1) The flow exiting into the air is steady, incompressible, and irrotational (so that the Bernoulli equation is applicable). 2) The water pressure in the hose near the outlet is equal to the water main pressure. 3) The surface tension effects are negligible. 4) The friction between the water and air is negligible. 5) The irreversibilities that may occur at the outlet of the hose due to abrupt expansion are negligible.

Properties: We take the density of water to be  $1000 \text{ kg/m}^3$ .

The velocity inside the hose is relatively low ( $V_1 = 0$ ) and we take the hose outlet as the reference level ( $z_1 = 0$ ). At the top of the water trajectory  $V_2 = 0$ , and atmospheric pressure pertains. Then the Bernoulli equation simplifies to,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \rightarrow \frac{P_1}{\rho g} = \frac{P_{\text{atm}}}{\rho g} + z_2$$

$$z_2 = \frac{P_1 - P_{\text{atm}}}{\rho g} = \frac{P_{1, \text{gage}}}{\rho g} = \frac{400 \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1000 \text{ N/m}^2}{1 \text{ kPa}} \right) \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right)$$

$$= 40.8 \text{ m}$$

Discussion: It tells us that the water cannot possibly rise more than 40.8 m, and, in all likelihood, the rise will be much less than 40.8 m due to irreversible losses that we neglected.

**Example 6:** A large tank open to the atmosphere is filled with water to a height of 5 m from the outlet tap. A tap near the bottom of the tank is now opened, and water flows out from the smooth and rounded outlet. Determine the water velocity at the outlet.

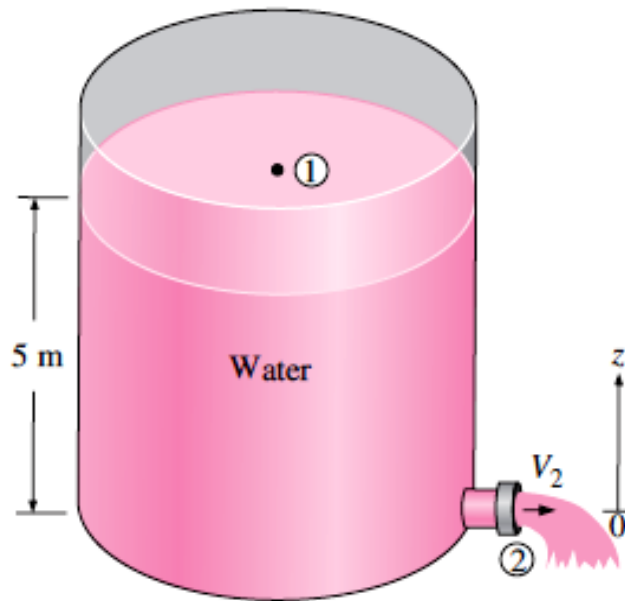


Figure 4-13.

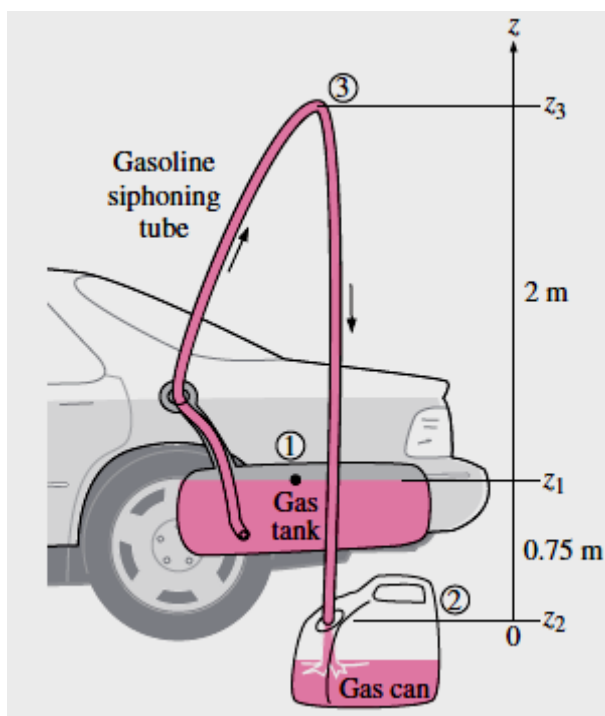
**Solution:**

Assumptions: 1) The flow is incompressible and irrotational 2) The water drains slowly enough that the flow can be approximated as steady.

$$\frac{\cancel{P_1}}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{\cancel{P_2}}{\rho g} + \frac{V_2^2}{2g} + z_2 \rightarrow z_1 = \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81 \text{ m/s}^2)(5 \text{ m})} = 9.9 \text{ m/s}$$

**Example 7:** During a trip to the beach ( $P_{\text{atm}} = 1 \text{ atm}$ ,  $\rho = 101.3 \text{ kPa}$ ), a car runs out of gasoline, and it becomes necessary to siphon gas out of the car of a Good Samaritan. The siphon is a small-diameter hose, and to start the siphon it is necessary to insert one siphon end in the full gas tank, fill the hose with gasoline via suction, and then place the other end in a gas can below the level of the gas tank. The difference in pressure between point 1 (at the free surface of the gasoline in the tank) and point 2 (at the outlet of the tube) causes the liquid to flow from the higher to the lower elevation. Point 2 is located 0.75 m below point 1 in this case, and point 3 is located 2 m above point 1. The siphon diameter is 4 mm, and frictional losses in the siphon are to be disregarded. Determine (a) the minimum time to withdraw 4 L of gasoline from the tank to the can and (b) the pressure at point 3. The density of gasoline is  $750 \text{ kg/m}^3$ .



**SOLUTION** Gasoline is to be siphoned from a tank. The minimum time it takes to withdraw 4 L of gasoline and the pressure at the highest point in the system are to be determined.

**Assumptions** 1 The flow is steady and incompressible. 2 Even though the Bernoulli equation is not valid through the pipe because of frictional losses, we employ the Bernoulli equation anyway in order to obtain a *best-case estimate*. 3 The change in the gasoline surface level inside the tank is negligible compared to elevations  $z_1$  and  $z_2$  during the siphoning period.

**Properties** The density of gasoline is given to be  $750 \text{ kg/m}^3$ .

**Analysis** (a) We take point 1 to be at the free surface of gasoline in the tank so that  $P_1 = P_{\text{atm}}$  (open to the atmosphere),  $V_1 \cong 0$  (the tank is large relative to the tube diameter), and  $z_2 = 0$  (point 2 is taken as the reference level). Also,  $P_2 = P_{\text{atm}}$  (gasoline discharges into the atmosphere). Then the Bernoulli equation simplifies to

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + z_2 \rightarrow z_1 = \frac{V_2^2}{2g}$$

Solving for  $V_2$  and substituting,

$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81 \text{ m/s}^2)(0.75 \text{ m})} = 3.84 \text{ m/s}$$

The cross-sectional area of the tube and the flow rate of gasoline are

$$A = \pi D^2/4 = \pi(5 \times 10^{-3} \text{ m})^2/4 = 1.96 \times 10^{-5} \text{ m}^2$$

$$\dot{V} = V_2 A = (3.84 \text{ m/s})(1.96 \times 10^{-5} \text{ m}^2) = 7.53 \times 10^{-5} \text{ m}^3/\text{s} = 0.0753 \text{ L/s}$$

Then the time needed to siphon 4 L of gasoline becomes

$$\Delta t = \frac{V}{\dot{V}} = \frac{4 \text{ L}}{0.0753 \text{ L/s}} = \mathbf{53.1 \text{ s}}$$

(b) The pressure at point 3 can be determined by writing the Bernoulli equation between points 2 and 3. Noting that  $V_2 = V_3$  (conservation of mass),  $z_2 = 0$ , and  $P_2 = P_{\text{atm}}$ ,

$$\frac{P_2}{\rho g} + \frac{\cancel{V_2^2}}{\cancel{2g}} + z_2 \overset{0}{=} \frac{P_3}{\rho g} + \frac{\cancel{V_3^2}}{\cancel{2g}} + z_3 \rightarrow \frac{P_{\text{atm}}}{\rho g} = \frac{P_3}{\rho g} + z_3$$

Solving for  $P_3$  and substituting,

$$\begin{aligned} P_3 &= P_{\text{atm}} - \rho g z_3 \\ &= 101.3 \text{ kPa} - (750 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.75 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kPa}}{1000 \text{ N/m}^2} \right) \\ &= \mathbf{81.1 \text{ kPa}} \end{aligned}$$

**Example 8:** A piezometer and a Pitot tube are tapped into a horizontal water pipe to measure static and stagnation (static + dynamic) pressures. For the indicated water column heights, determine the velocity at the center of the pipe.

**Solution:**

Assumptions: 1) The flow is steady and incompressible. 2) Points 1 and 2 are close enough together that the irreversible energy loss between these two points is negligible, and thus we can use the Bernoulli equation.

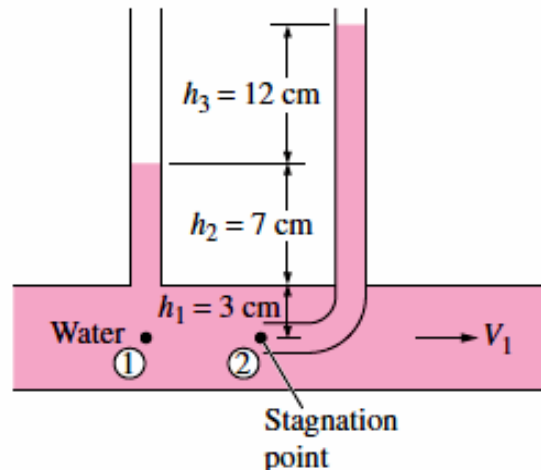


Figure 4-14.

$$P_1 = \rho g(h_1 + h_2)$$

$$P_2 = \rho g(h_1 + h_2 + h_3)$$

Noting that point 2 is a stagnation point and thus  $V_2 = 0$  and  $z_1 = z_2$ , the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2} \rightarrow \frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g}$$

Substituting the  $P_1$  and  $P_2$  expressions gives

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g} = \frac{\rho g(h_1 + h_2 + h_3) - \rho g(h_1 + h_2)}{\rho g} = h_3$$

Solving for  $V_1$  and substituting,

$$V_1 = \sqrt{2gh_3} = \sqrt{2(9.81 \text{ m/s}^2)(0.12 \text{ m})} = \mathbf{1.53 \text{ m/s}}$$

**Example 9:** A pressurized tank of water has a 10-cm-diameter orifice at the bottom, where water discharges to the atmosphere. The water level is 3 m above the outlet. The tank air pressure above the water level is 300 kPa (absolute) while the atmospheric pressure is 100 kPa. Neglecting frictional effects, determine the initial discharge rate of water from the tank.

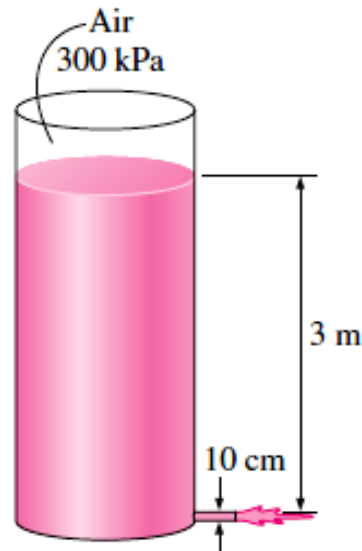


Figure 4-15.

**Assumptions** 1 The orifice has a smooth entrance, and thus the frictional losses are negligible. 2 The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

**Properties** We take the density of water to be  $1000 \text{ kg/m}^3$ .

**Analysis** We take point 1 at the free surface of the tank, and point 2 at the exit of orifice, which is also taken to be the reference level ( $z_2 = 0$ ). Noting that the fluid velocity at the free surface is very low ( $V_1 \cong 0$ ) and water discharges into the atmosphere (and thus  $P_2 = P_{\text{atm}}$ ), the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \rightarrow \frac{V_2^2}{2g} = \frac{P_1 - P_2}{\rho g} + z_1$$

Solving for  $V_2$  and substituting, the discharge velocity is determined to

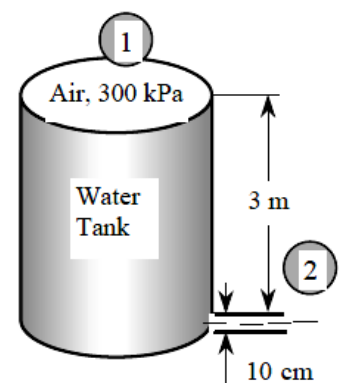
$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho} + 2gz_1} = \sqrt{\frac{2(300 - 100) \text{ kPa}}{1000 \text{ kg/m}^3} \left( \frac{1000 \text{ N/m}^2}{1 \text{ kPa}} \right) \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) + 2(9.81 \text{ m/s}^2)(3 \text{ m})}$$

$$= 21.4 \text{ m/s}$$

Then the initial rate of discharge of water becomes

$$\dot{V} = A_{\text{orifice}} V_2 = \frac{\pi D^2}{4} V_2 = \frac{\pi (0.10 \text{ m})^2}{4} (21.4 \text{ m/s}) = \mathbf{0.168 \text{ m}^3/\text{s}}$$

**Discussion** Note that this is the maximum flow rate since the frictional effects are ignored. Also, the velocity and the flow rate will decrease as the water level in the tank decreases.



**Example 10:** The water in a 10-m-diameter, 2-m-high aboveground swimming pool is to be emptied by unplugging a 3-cm-diameter, 25-m-long horizontal pipe attached to the bottom of the pool. Determine the maximum discharge rate of water through the pipe. Also, explain why the actual flow rate will be less.

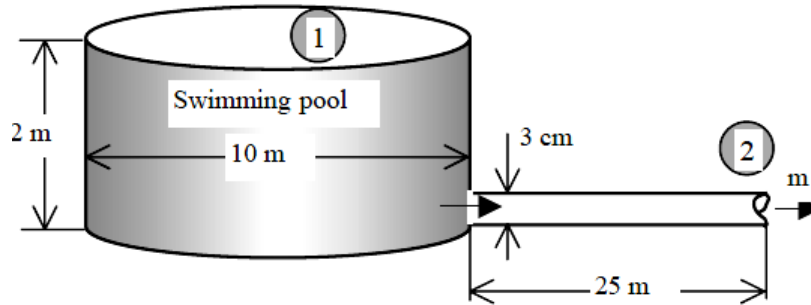


Figure 4-16.

**Assumptions** 1 The orifice has a smooth entrance, and all frictional losses are negligible. 2 The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

**Analysis** We take point 1 at the free surface of the pool, and point 2 at the exit of pipe. We take the reference level at the pipe exit ( $z_2 = 0$ ). Noting that the fluid at both points is open to the atmosphere (and thus  $P_1 = P_2 = P_{\text{atm}}$ ) and that the fluid velocity at the free surface is very low ( $V_1 \cong 0$ ), the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \rightarrow z_1 = \frac{V_2^2}{2g} \rightarrow V_2 = \sqrt{2gz_1}$$

The maximum discharge rate occurs when the water height in the pool is a maximum, which is the case at the beginning and thus  $z_1 = h$ . Substituting, the maximum flow velocity and discharge rate become

$$V_{2,\text{max}} = \sqrt{2gh} = \sqrt{2(9.81 \text{ m/s}^2)(2 \text{ m})} = 6.26 \text{ m/s}$$

$$\dot{V}_{\text{max}} = A_{\text{pipe}} V_{2,\text{max}} = \frac{\pi D^2}{4} V_{2,\text{max}} = \frac{\pi (0.03 \text{ m})^2}{4} (6.26 \text{ m/s}) = 0.00443 \text{ m}^3/\text{s} = \mathbf{4.43 \text{ L/s}}$$

**Discussion** The result above is obtained by disregarding all frictional effects. The actual flow rate will be less because of frictional effects during flow and the resulting pressure drop. Also, the flow rate will gradually decrease as the water level in the pool decreases.

**Example 11:** The water level in a tank is 20 m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank cover is airtight, and the air pressure above the water surface is 2 atm gage. The system is at sea level. Determine the maximum height to which the water stream could rise.

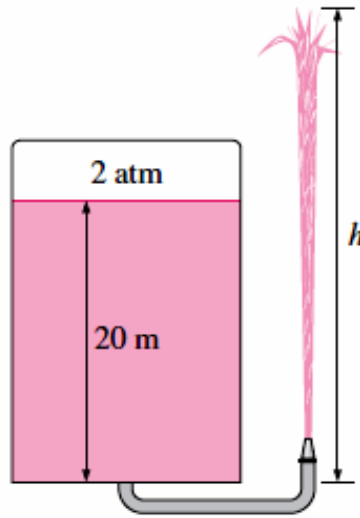


Figure 4-17.

**Assumptions 1** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). **2** The friction between the water and air is negligible.

**Properties** We take the density of water to be  $1000 \text{ kg/m}^3$ .

**Analysis** We take point 1 at the free surface of water in the tank, and point 2 at the top of the water trajectory. Also, we take the reference level at the bottom of the tank. At the top of the water trajectory  $V_2 = 0$ , and atmospheric pressure pertains. Noting that  $z_1 = 20 \text{ m}$ ,  $P_{1,\text{gage}} = 2 \text{ atm}$ ,  $P_2 = P_{\text{atm}}$ , and that the fluid velocity at the free surface of the tank is very low ( $V_1 \cong 0$ ), the Bernoulli equation between these two points simplifies to

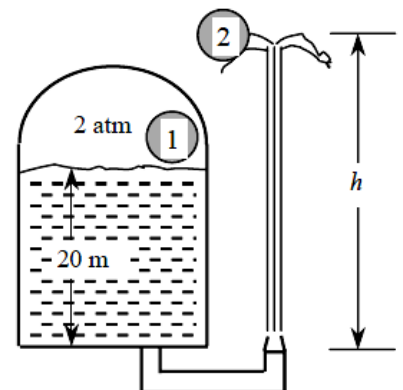
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \rightarrow \frac{P_1}{\rho g} + z_1 = \frac{P_{\text{atm}}}{\rho g} + z_2 \rightarrow z_2 = \frac{P_1 - P_{\text{atm}}}{\rho g} + z_1 = \frac{P_{1,\text{gage}}}{\rho g} + z_1$$

Substituting,

$$z_2 = \frac{2 \text{ atm}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{101,325 \text{ N/m}^2}{1 \text{ atm}} \right) \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) + 20 = 40.7 \text{ m}$$

Therefore, the water jet can rise as high as 40.7 m into the sky from the ground.

**Discussion** The result obtained by the Bernoulli equation represents the upper limit, and should be interpreted accordingly. It tells us that the water cannot possibly rise more than 40.7 m (giving us an upper limit), and in all likelihood, the rise will be much less because of frictional losses.



**Example 12:** A Pitot-static probe is used to measure the velocity of an aircraft flying at 3000 m. If the differential pressure reading is 3 kPa, determine the velocity of the aircraft.

**Assumptions** 1 The air flow over the aircraft is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). 2 Standard atmospheric conditions exist. 3 The wind effects are negligible.

**Properties** The density of the atmosphere at an elevation of 3000 m is  $\rho = 0.909 \text{ kg/m}^3$ .

**Analysis** We take point 1 at the entrance of the tube whose opening is parallel to flow, and point 2 at the entrance of the tube whose entrance is normal to flow. Noting that point 2 is a stagnation point and thus  $V_2 = 0$  and  $z_1 = z_2$ , the application of the Bernoulli equation between points 1 and 2 gives

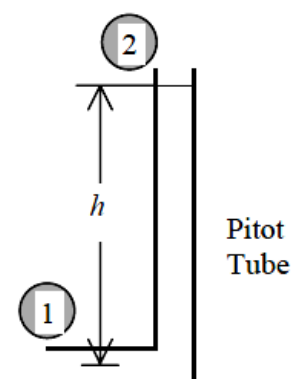
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \rightarrow \frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g} \rightarrow \frac{V_1^2}{2} = \frac{P_{stag} - P_1}{\rho}$$

Solving for  $V_1$  and substituting,

$$V_1 = \sqrt{\frac{2(P_{stag} - P_1)}{\rho}} = \sqrt{\frac{2(3000 \text{ N/m}^2) \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right)}{0.909 \text{ kg/m}^3}} = 81.2 \text{ m/s} = 292 \text{ km/h}$$

since  $1 \text{ Pa} = 1 \text{ N/m}^2$  and  $1 \text{ m/s} = 3.6 \text{ km/h}$ .

**Discussion** Note that the velocity of an aircraft can be determined by simply measuring the differential pressure on a Pitot-static probe.



**Example 13:** While traveling on a dirt road, the bottom of a car hits a sharp rock and a small hole develops at the bottom of its gas tank. If the height of the gasoline in the tank is 30 cm, determine the initial velocity of the gasoline at the hole. Discuss how the velocity will change with time and how the flow will be affected if the lid of the tank is closed tightly.

**Assumptions** 1 The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). 2 The air space in the tank is at atmospheric pressure. 3 The splashing of the gasoline in the tank during travel is not considered.

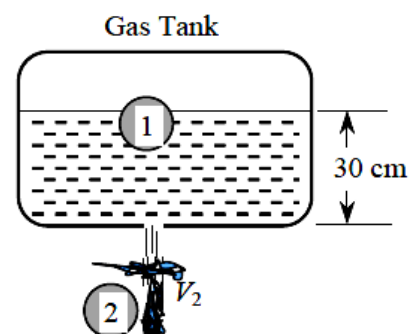
**Analysis** This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. We take point 1 to be at the free surface of gasoline in the tank so that  $P_1 = P_{atm}$  (open to the atmosphere)  $V_1 \cong 0$  (the tank is large relative to the outlet), and  $z_1 = 0.3 \text{ m}$  and  $z_2 = 0$  (we take the reference level at the hole). Also,  $P_2 = P_{atm}$  (gasoline discharges into the atmosphere). Then the Bernoulli Equation simplifies to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \rightarrow z_1 = \frac{V_2^2}{2g}$$

Solving for  $V_2$  and substituting,

$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81 \text{ m/s}^2)(0.3 \text{ m})} = 2.43 \text{ m/s}$$

Therefore, the gasoline will initially leave the tank with a velocity of 2.43 m/s.



**Discussion** The Bernoulli equation applies along a streamline, and streamlines generally do not make sharp turns. The velocity will be less than 2.43 m/s since the hole is probably sharp-edged and it will cause some head loss.

As the gasoline level is reduced, the velocity will decrease since velocity is proportional to the square root of liquid height. If the lid is tightly closed and no air can replace the lost gasoline volume, the pressure above the gasoline level will be reduced, and the velocity will be decreased.

**Example 14:** Air at 110 kPa and 50°C flows upward through a 6-cm-diameter inclined duct at a rate of 45 L/s. The duct diameter is then reduced to 4 cm through a reducer. The pressure change across the reducer is measured by a water manometer. The elevation difference between the two points on the pipe where the two arms of the manometer are attached is 0.20 m. Determine the differential height between the fluid levels of the two arms of the manometer.

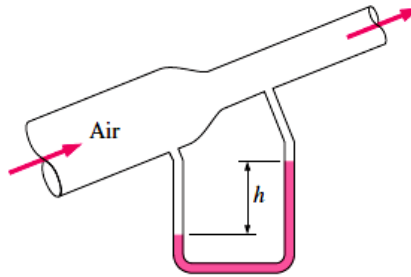


Figure 4-18.

**Assumptions** 1 The flow through the duct is steady, incompressible and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). 2 Air is an ideal gas. 3 The effect of air column on the pressure change is negligible because of its low density. 3 The air flow is parallel to the entrance of each arm of the manometer, and thus no dynamic effects are involved.

**Properties** We take the density of water to be  $\rho = 1000 \text{ kg/m}^3$ . The gas constant of air is  $R = 0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$ .

**Analysis** We take points 1 and 2 at the lower and upper connection points, respectively, of the two arms of the manometer, and take the lower connection point as the reference level. Noting that the effect of elevation on the pressure change of a gas is negligible, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \rightarrow P_1 - P_2 = \rho_{\text{air}} \frac{V_2^2 - V_1^2}{2}$$

where  $\rho_{\text{air}} = \frac{P}{RT} = \frac{110 \text{ kPa}}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(50 + 273 \text{ K})} = 1.19 \text{ kg/m}^3$

$$V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\pi D_1^2 / 4} = \frac{0.045 \text{ m}^3/\text{s}}{\pi (0.06 \text{ m})^2 / 4} = 15.9 \text{ m/s}$$

$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2 / 4} = \frac{0.045 \text{ m}^3/\text{s}}{\pi (0.04 \text{ m})^2 / 4} = 35.8 \text{ m/s}$$

Substituting,

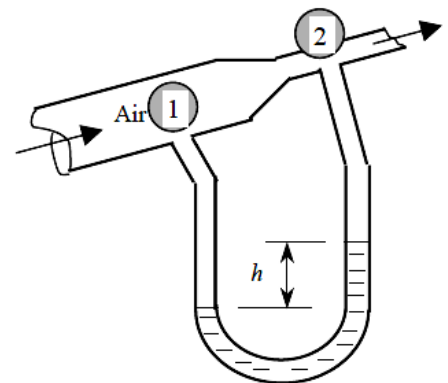
$$P_1 - P_2 = (1.19 \text{ kg/m}^3) \frac{(35.8 \text{ m/s})^2 - (15.9 \text{ m/s})^2}{2} \left( \frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2} \right) = 612 \text{ N/m}^2 = 612 \text{ Pa}$$

The differential height of water in the manometer corresponding to this pressure change is determined from  $\Delta P = \rho_{\text{water}} g h$  to be

$$h = \frac{P_1 - P_2}{\rho_{\text{water}} g} = \frac{612 \text{ N/m}^2}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}} \right) = 0.0624 \text{ m} = \mathbf{6.24 \text{ cm}}$$

**Discussion** When the effect of air column on pressure change is considered, the pressure change becomes

$$\begin{aligned} P_1 - P_2 &= \frac{\rho_{\text{air}} (V_2^2 - V_1^2)}{2} + \rho_{\text{air}} g (z_2 - z_1) \\ &= (1.19 \text{ kg/m}^3) \left[ \frac{(35.8 \text{ m/s})^2 - (15.9 \text{ m/s})^2}{2} + (9.81 \text{ m/s}^2)(0.2 \text{ m}) \right] \left( \frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2} \right) \\ &= (612 + 2) \text{ N/m}^2 = 614 \text{ N/m}^2 = 614 \text{ Pa} \end{aligned}$$



**Example 15:** A Pitot-static probe connected to a water manometer is used to measure the velocity of air. If the deflection (the vertical distance between the fluid levels in the two arms) is 7.3 cm, determine the air velocity. Take the density of air to be  $1.25 \text{ kg/m}^3$ .

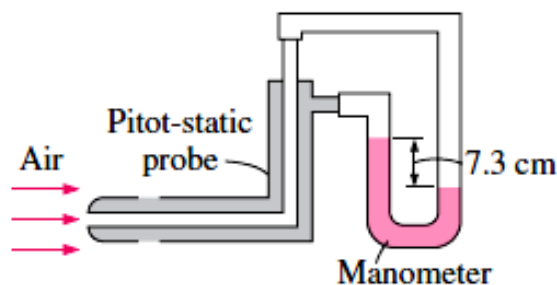


Figure 4-19.

**Assumptions** 1 The flow of air is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). 2 The effect of air column on the pressure change is negligible because of its low density, and thus the air column in the manometer can be ignored.

**Properties** We take the density of water to be  $\rho = 1000 \text{ kg/m}^3$ . The density of air is given to be  $1.25 \text{ kg/m}^3$ .

**Analysis** We take point 1 on the side of the probe where the entrance is parallel to flow and is connected to the static arm of the Pitot-static probe, and point 2 at the tip of the probe where the entrance is normal to flow and is connected to the dynamic arm of the Pitot-static probe. Noting that point 2 is a stagnation point and thus  $V_2 = 0$  and  $z_1 = z_2$ , the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} \rightarrow V_1 = \sqrt{\frac{2(P_2 - P_1)}{\rho_{\text{air}}}} \quad (1)$$

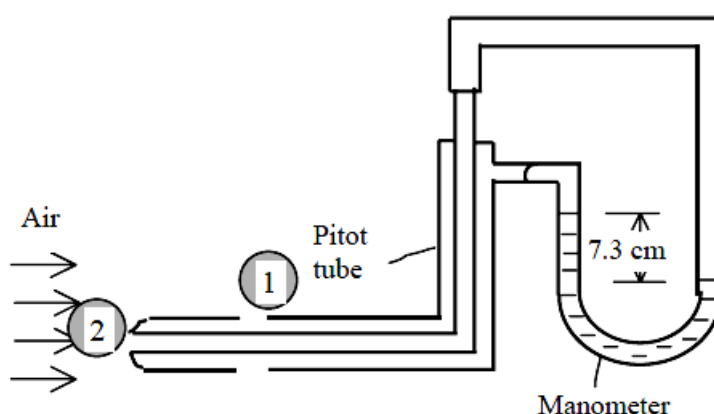
The pressure rise at the tip of the Pitot-static probe is simply the pressure change indicated by the differential water column of the manometer,

$$P_2 - P_1 = \rho_{\text{water}} g h \quad (2)$$

Combining Eqs. (1) and (2) and substituting, the flow velocity is determined to be

$$V_1 = \sqrt{\frac{2\rho_{\text{water}} g h}{\rho_{\text{air}}}} = \sqrt{\frac{2(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.073 \text{ m})}{1.25 \text{ kg/m}^3}} = 33.8 \text{ m/s}$$

**Discussion** Note that flow velocity in a pipe or duct can be measured easily by a Pitot-static probe by inserting the probe into the pipe or duct parallel to flow, and reading the differential height. Also note that this is the velocity at the location of the tube. Several readings at several locations in a cross-section may be required to determine the mean flow velocity.



**Example 16:** The pump of a water distribution system is powered by a 15-kW electric motor whose efficiency is 90 percent. The water flow rate through the pump is 50 L/s. The diameters of the inlet and outlet pipes are the same, and the elevation difference across the pump is negligible. If the pressures at the inlet and outlet of the pump are measured to be 100 kPa and 300 kPa (absolute), respectively, determine (a) the mechanical efficiency of the pump and (b) the temperature rise of water as it flows through the pump due to the mechanical inefficiency.

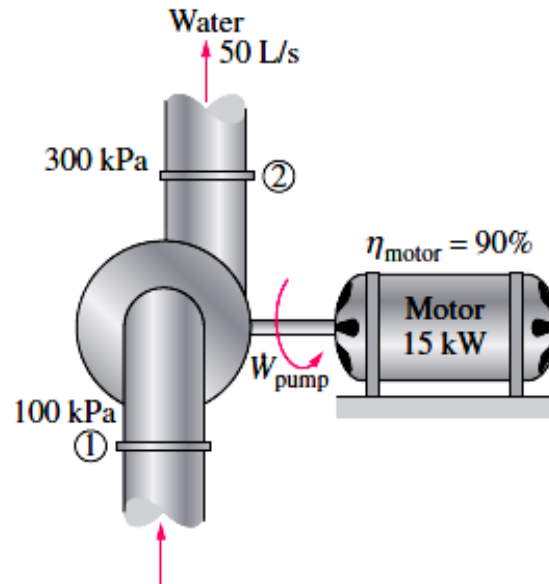


Figure 4-21

**Assumptions** 1 The flow is steady and incompressible. 2 The pump is driven by an external motor so that the heat generated by the motor is dissipated to the atmosphere. 3 The elevation difference between the inlet and outlet of the pump is negligible,  $z_1 \cong z_2$ . 4 The inlet and outlet diameters are the same and thus the inlet and outlet velocities and kinetic energy correction factors are equal,  $V_1 = V_2$  and  $\alpha_1 = \alpha_2$ .

**Properties** We take the density of water to be  $1 \text{ kg/L} = 1000 \text{ kg/m}^3$  and its specific heat to be  $4.18 \text{ kJ/kg} \cdot ^\circ\text{C}$ .

**Analysis** (a) The mass flow rate of water through the pump is

$$\dot{m} = \rho \dot{V} = (1 \text{ kg/L})(50 \text{ L/s}) = 50 \text{ kg/s}$$

The motor draws 15 kW of power and is 90 percent efficient. Thus the mechanical (shaft) power it delivers to the pump is

$$\dot{W}_{\text{pump, shaft}} = \eta_{\text{motor}} \dot{W}_{\text{electric}} = (0.90)(15 \text{ kW}) = 13.5 \text{ kW}$$

To determine the mechanical efficiency of the pump, we need to know the increase in the mechanical energy of the fluid as it flows through the pump, which is

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{E}_{\text{mech, out}} - \dot{E}_{\text{mech, in}} = \dot{m} \left( \frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2 \right) - \dot{m} \left( \frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1 \right)$$

Simplifying it for this case and substituting the given values,

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{m} \left( \frac{P_2 - P_1}{\rho} \right) = (50 \text{ kg/s}) \left( \frac{(300 - 100) \text{ kPa}}{1000 \text{ kg/m}^3} \right) \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) = 10 \text{ kW}$$

Then the mechanical efficiency of the pump becomes

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{pump, shaft}}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{pump, shaft}}} = \frac{10 \text{ kW}}{13.5 \text{ kW}} = \mathbf{0.741} \quad \text{or} \quad \mathbf{74.1\%}$$

(b) Of the 13.5-kW mechanical power supplied by the pump, only 10 kW is imparted to the fluid as mechanical energy. The remaining 3.5 kW is converted to thermal energy due to frictional effects, and this “lost” mechanical energy manifests itself as a heating effect in the fluid,

$$\dot{E}_{\text{mech, loss}} = \dot{W}_{\text{pump, shaft}} - \Delta \dot{E}_{\text{mech, fluid}} = 13.5 - 10 = 3.5 \text{ kW}$$

The temperature rise of water due to this mechanical inefficiency is determined from the thermal energy balance,  $\dot{E}_{\text{mech, loss}} = \dot{m}(u_2 - u_1) = \dot{m}c\Delta T$ . Solving for  $\Delta T$ ,

$$\Delta T = \frac{\dot{E}_{\text{mech, loss}}}{\dot{m}c} = \frac{3.5 \text{ kW}}{(50 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot ^\circ\text{C})} = \mathbf{0.017^\circ\text{C}}$$

Therefore, the water will experience a temperature rise of 0.017°C due to mechanical inefficiency, which is very small, as it flows through the pump.

**Example 17:** In a hydroelectric power plant, 100 m<sup>3</sup>/s of water flows from an elevation of 120 m to a turbine, where electric power is generated. The total irreversible head loss in the piping system from point 1 to point 2 (excluding the turbine unit) is determined to be 35 m. If the overall efficiency of the turbine-generator is 80 percent, estimate the electric power output.

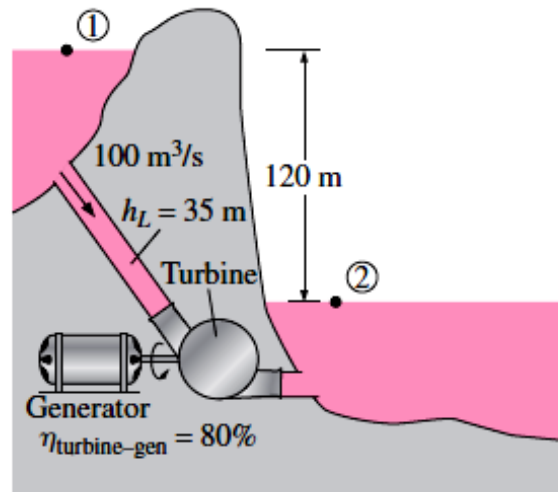


Figure 4-22

**Solution:** The available head, flow rate, head loss, and efficiency of a hydroelectric turbine are given. The electric power output is to be determined.

Assumptions: 1) The flow is steady and incompressible. 2) Water levels at the reservoir and the discharge site remain constant.

Properties: We take the density of water to be  $1000 \text{ kg/m}^3$ .

**Analysis** The mass flow rate of water through the turbine is

$$\dot{m} = \rho \dot{V} = (1000 \text{ kg/m}^3)(100 \text{ m}^3/\text{s}) = 10^5 \text{ kg/s}$$

We take point 2 as the reference level, and thus  $z_2 = 0$ . Also, both points 1 and 2 are open to the atmosphere ( $P_1 = P_2 = P_{\text{atm}}$ ) and the flow velocities are negligible at both points ( $V_1 = V_2 = 0$ ). Then the energy equation for steady, incompressible flow reduces to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump},u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine},e} + h_L \rightarrow$$

$$h_{\text{turbine},e} = z_1 - h_L$$

Substituting, the extracted turbine head and the corresponding turbine power are

$$h_{\text{turbine},e} = z_1 - h_L = 120 - 35 = 85 \text{ m}$$

$$\dot{W}_{\text{turbine},e} = \dot{m}gh_{\text{turbine},e} = (10^5 \text{ kg/s})(9.81 \text{ m/s}^2)(85 \text{ m})\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 83,400 \text{ kW}$$

Therefore, a perfect turbine-generator would generate 83,400 kW of electricity from this resource. The electric power generated by the actual unit is

$$\dot{W}_{\text{electric}} = \eta_{\text{turbine-gen}} \dot{W}_{\text{turbine},e} = (0.80)(83.4 \text{ MW}) = 66.7 \text{ MW}$$

**Discussion** Note that the power generation would increase by almost 1 MW for each percentage point improvement in the efficiency of the turbine-generator unit.

**Example 18:** Water is pumped from a lower reservoir to a higher reservoir by a pump that provides 20 kW of useful mechanical power to the water. The free surface of the upper reservoir is 45 m higher than the surface of the lower reservoir. If the flow rate of water is measured to be  $0.03 \text{ m}^3/\text{s}$ , determine the irreversible head loss of the system and the lost mechanical power during this process.

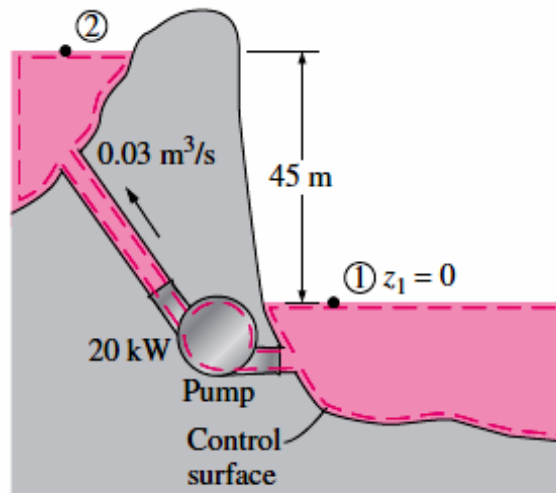


Figure 4-23

**SOLUTION** Water is pumped from a lower reservoir to a higher one. The head and power loss associated with this process are to be determined.

**Assumptions** 1 The flow is steady and incompressible. 2 The elevation difference between the reservoirs is constant.

**Properties** We take the density of water to be  $1000 \text{ kg/m}^3$ .

**Analysis** The mass flow rate of water through the system is

$$\dot{m} = \rho \dot{V} = (1000 \text{ kg/m}^3)(0.03 \text{ m}^3/\text{s}) = 30 \text{ kg/s}$$

We choose points 1 and 2 at the free surfaces of the lower and upper reservoirs, respectively, and take the surface of the lower reservoir as the reference level ( $z_1 = 0$ ). Both points are open to the atmosphere ( $P_1 = P_2 = P_{\text{atm}}$ ) and the velocities at both locations are negligible ( $V_1 = V_2 = 0$ ). Then the energy equation for steady incompressible flow for a control volume between 1 and 2 reduces to

$$\begin{aligned} \dot{m} \left( \frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1 \right) + \dot{W}_{\text{pump}} \\ = \dot{m} \left( \frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2 \right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}} \end{aligned}$$

$$\dot{W}_{\text{pump}} = \dot{m}gz_2 + \dot{E}_{\text{mech, loss}} \quad \rightarrow \quad \dot{E}_{\text{mech, loss}} = \dot{W}_{\text{pump}} - \dot{m}gz_2$$

Substituting, the lost mechanical power and head loss are determined to be

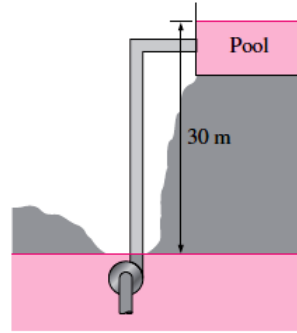
$$\begin{aligned} \dot{E}_{\text{mech, loss}} &= 20 \text{ kW} - (30 \text{ kg/s})(9.81 \text{ m/s}^2)(45 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kW}}{1000 \text{ N} \cdot \text{m/s}} \right) \\ &= \mathbf{6.76 \text{ kW}} \end{aligned}$$

Noting that the entire mechanical losses are due to frictional losses in piping and thus  $\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech, loss, piping}}$ , the irreversible head loss is determined to be

$$h_L = \frac{\dot{E}_{\text{mech, loss, piping}}}{\dot{m}g} = \frac{6.76 \text{ kW}}{(30 \text{ kg/s})(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) \left( \frac{1000 \text{ N} \cdot \text{m/s}}{1 \text{ kW}} \right) = \mathbf{23.0 \text{ m}}$$

**Example 19:** Underground water is to be pumped by a 70 percent efficient 3-kW submerged pump to a pool whose free surface is 30 m above the underground water level. The diameter of the pipe is 7 cm on the intake side and 5 cm on the discharge side. Determine (a) the maximum flow rate of water and (b) the

pressure difference across the pump. Assume the elevation difference between the pump inlet and the outlet and the effect of the kinetic energy correction factors to be negligible.



**Figure 4-24**

**Assumptions** 1 The flow is steady and incompressible. 2 The elevation difference between the inlet and the outlet of the pump is negligible. 3 We assume the frictional effects in piping to be negligible since the maximum flow rate is to be determined,  $\dot{E}_{\text{mech,loss, piping}} = 0$ . 4 The effect of the kinetic energy correction factors is negligible,  $\alpha = 1$ .

**Properties** We take the density of water to be  $1 \text{ kg/L} = 1000 \text{ kg/m}^3$ .

**Analysis** (a) The pump-motor draws 3-kW of power, and is 70% efficient. Then the useful mechanical (shaft) power it delivers to the fluid is

$$\dot{W}_{\text{pump, u}} = \eta_{\text{pump-motor}} \dot{W}_{\text{electric}} = (0.70)(3 \text{ kW}) = 2.1 \text{ kW}$$

We take point 1 at the free surface of underground water, which is also taken as the reference level ( $z_1 = 0$ ), and point 2 at the free surface of the pool. Also, both 1 and 2 are open to the atmosphere ( $P_1 = P_2 = P_{\text{atm}}$ ), the velocities are negligible at both points ( $V_1 \cong V_2 \cong 0$ ), and frictional losses in piping are disregarded. Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\dot{m} \left( \frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1 \right) + \dot{W}_{\text{pump}} = \dot{m} \left( \frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2 \right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}}$$

In the absence of a turbine,  $\dot{E}_{\text{mech,loss}} = \dot{E}_{\text{mech,loss,pump}} + \dot{E}_{\text{mech,loss,piping}}$  and

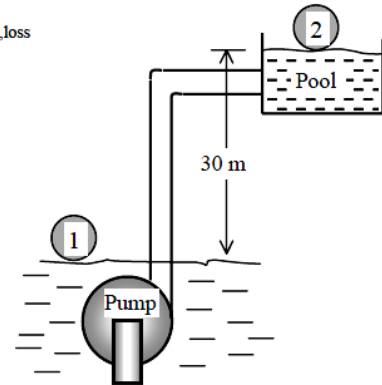
$\dot{W}_{\text{pump, u}} = \dot{W}_{\text{pump}} - \dot{E}_{\text{mech,loss,pump}}$ . Thus,

$$\dot{W}_{\text{pump, u}} = \dot{m}gz_2$$

Then the mass and volume flow rates of water become

$$\dot{m} = \frac{\dot{W}_{\text{pump, u}}}{gz_2} = \frac{2.1 \text{ kJ/s}}{(9.81 \text{ m/s}^2)(30 \text{ m})} \left( \frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ}} \right) = 7.14 \text{ kg/s}$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{7.14 \text{ kg/s}}{1000 \text{ kg/m}^3} = 7.14 \times 10^{-3} \text{ m}^3/\text{s}$$



(b) We take points 3 and 4 at the inlet and the exit of the pump, respectively, where the flow velocities are

$$V_3 = \frac{\dot{V}}{A_3} = \frac{\dot{V}}{\pi D_3^2 / 4} = \frac{7.14 \times 10^{-3} \text{ m}^3/\text{s}}{\pi (0.07 \text{ m})^2 / 4} = 1.86 \text{ m/s}, \quad V_4 = \frac{\dot{V}}{A_4} = \frac{\dot{V}}{\pi D_4^2 / 4} = \frac{7.14 \times 10^{-3} \text{ m}^3/\text{s}}{\pi (0.05 \text{ m})^2 / 4} = 3.64 \text{ m/s}$$

We take the pump as the control volume. Noting that  $z_3 = z_4$ , the energy equation for this control volume reduces to

$$\dot{m} \left( \frac{P_3}{\rho} + \alpha_3 \frac{V_3^2}{2} + gz_3 \right) + \dot{W}_{\text{pump}} = \dot{m} \left( \frac{P_4}{\rho} + \alpha_4 \frac{V_4^2}{2} + gz_4 \right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss,pump}}$$

$$\rightarrow P_4 - P_3 = \frac{\rho \alpha (V_3^2 - V_4^2)}{2} + \frac{\dot{W}_{\text{pump, u}}}{\dot{V}}$$

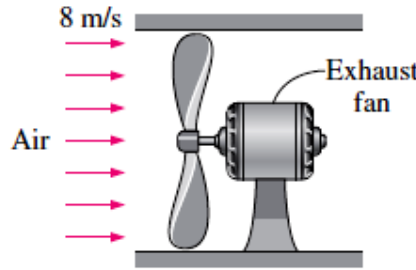
Substituting,

$$P_4 - P_3 = \frac{(1000 \text{ kg/m}^3)(1.0)[(1.86 \text{ m/s})^2 - (3.64 \text{ m/s})^2]}{2} \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) + \frac{2.1 \text{ kJ/s}}{7.14 \times 10^{-3} \text{ m}^3/\text{s}} \left( \frac{1 \text{ kN} \cdot \text{m}}{1 \text{ kJ}} \right)$$

$$= (-4.9 + 294.1) \text{ kN/m}^2 = \mathbf{289.2 \text{ kPa}}$$

**Example 20:** A fan is to be selected to ventilate a bathroom whose dimensions are 2 m x 3 m x 3 m. The air velocity is not to exceed 8 m/s to minimize vibration and noise. The combined efficiency of the fan-motor unit to be used can be taken to be 50 percent. If the fan is to replace the entire volume of air in 10 min, determine

(a) the wattage of the fan–motor unit to be purchased, (b) the diameter of the fan casing, and (c) the pressure difference across the fan. Take the air density to be  $1.25 \text{ kg/m}^3$  and disregard the effect of the kinetic energy correction factors.



**Assumptions** 1 The flow is steady and incompressible. 2 Frictional losses along the flow (other than those due to the fan-motor inefficiency) are negligible. 3 The fan unit is horizontal so that  $z = \text{constant}$  along the flow (or, the elevation effects are negligible because of the low density of air). 4 The effect of the kinetic energy correction factors is negligible,  $\alpha = 1$ .

**Properties** The density of air is given to be  $1.25 \text{ kg/m}^3$ .

**Analysis** (a) The volume of air in the bathroom is  $V = 2 \text{ m} \times 3 \text{ m} \times 3 \text{ m} = 18 \text{ m}^3$ . Then the volume and mass flow rates of air through the casing must be

$$\dot{V} = \frac{V}{\Delta t} = \frac{18 \text{ m}^3}{10 \times 60 \text{ s}} = 0.03 \text{ m}^3/\text{s}$$

$$\dot{m} = \rho \dot{V} = (1.25 \text{ kg/m}^3)(0.03 \text{ m}^3/\text{s}) = 0.0375 \text{ kg/s}$$

We take points 1 and 2 on the inlet and exit sides of the fan, respectively. Point 1 is sufficiently far from the fan so that  $P_1 = P_{\text{atm}}$  and the flow velocity is negligible ( $V_1 = 0$ ). Also,  $P_2 = P_{\text{atm}}$ . Then the energy equation for this control volume between the points 1 and 2 reduces to

$$\dot{m} \left( \frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1 \right) + \dot{W}_{\text{pump}} = \dot{m} \left( \frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2 \right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \rightarrow \dot{W}_{\text{fan,u}} = \dot{m} \alpha_2 \frac{V_2^2}{2}$$

since  $\dot{E}_{\text{mech,loss}} = \dot{E}_{\text{mech,loss,pump}}$  in this case and  $\dot{W}_{\text{pump,u}} = \dot{W}_{\text{pump}} - \dot{E}_{\text{mech,loss,pump}}$ . Substituting,

$$\dot{W}_{\text{fan,u}} = \dot{m} \alpha_2 \frac{V_2^2}{2} = (0.0375 \text{ kg/s})(1.0) \frac{(8 \text{ m/s})^2}{2} \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ W}}{1 \text{ N} \cdot \text{m/s}} \right) = 1.2 \text{ W}$$

and  $\dot{W}_{\text{fan,elect}} = \frac{\dot{W}_{\text{fan,u}}}{\eta_{\text{fan-motor}}} = \frac{1.2 \text{ W}}{0.5} = \mathbf{2.4 \text{ W}}$

Therefore, the electric power rating of the fan/motor unit must be 2.4 W.

(b) For air mean velocity to remain below the specified value, the diameter of the fan casing should be

$$\dot{V} = A_2 V_2 = (\pi D_2^2 / 4) V_2 \rightarrow D_2 = \sqrt{\frac{4 \dot{V}}{\pi V_2}} = \sqrt{\frac{4(0.03 \text{ m}^3/\text{s})}{\pi(8 \text{ m/s})}} = 0.069 \text{ m} = \mathbf{6.9 \text{ cm}}$$

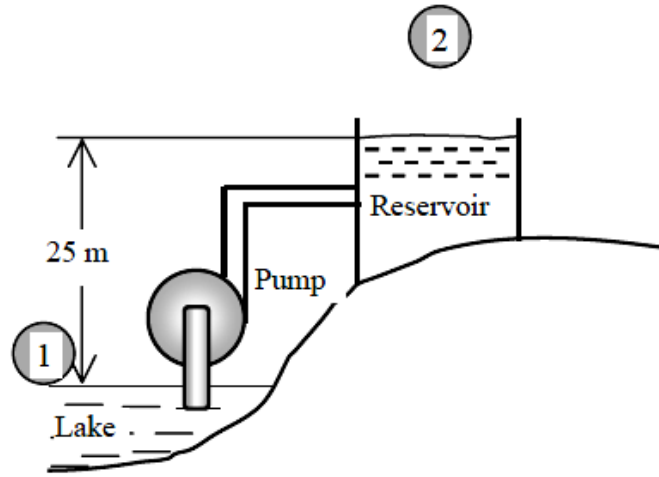
(c) To determine the pressure difference across the fan unit, we take points 3 and 4 to be on the two sides of the fan on a horizontal line. Noting that  $z_3 = z_4$  and  $V_3 = V_4$  since the fan is a narrow cross-section and neglecting flow losses (other than the losses of the fan unit, which is accounted for by the efficiency), the energy equation for the fan section reduces to

$$\dot{m} \frac{P_3}{\rho} + \dot{W}_{\text{fan,u}} = \dot{m} \frac{P_4}{\rho} \rightarrow P_4 - P_3 = \frac{\dot{W}_{\text{fan,u}}}{\dot{m} / \rho} = \frac{\dot{W}_{\text{fan,u}}}{\dot{V}}$$

Substituting,  $P_4 - P_3 = \frac{1.2 \text{ W}}{0.03 \text{ m}^3/\text{s}} \left( \frac{1 \text{ N} \cdot \text{m/s}}{1 \text{ W}} \right) = 40 \text{ N/m}^2 = \mathbf{40 \text{ Pa}}$

Therefore, the fan will raise the pressure of air by 40 Pa before discharging it.

**Example 21:** Water is being pumped from a large lake to a reservoir 25 m above at a rate of 25 L/s by a 10-kW (shaft) pump. If the irreversible head loss of the piping system is 7 m, determine the mechanical efficiency of the pump.



**Assumptions** 1 The flow is steady and incompressible. 2 The elevation difference between the lake and the reservoir is constant.

**Properties** We take the density of water to be  $\rho = 1000 \text{ kg/m}^3$ .

**Analysis** We choose points 1 and 2 at the free surfaces of the lake and the reservoir, respectively, and take the surface of the lake as the reference level ( $z_1 = 0$ ). Both points are open to the atmosphere ( $P_1 = P_2 = P_{\text{atm}}$ ) and the velocities at both locations are negligible ( $V_1 = V_2 = 0$ ). Then the energy equation for steady incompressible flow through a control volume between these two points that includes the pump and the pipes reduces to

$$\dot{m} \left( \frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1 \right) + \dot{W}_{\text{pump}} = \dot{m} \left( \frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2 \right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \rightarrow$$

$$\dot{W}_{\text{pump,u}} = \dot{m}gz_2 + \dot{E}_{\text{mech,loss,piping}}$$

since, in the absence of a turbine,  $\dot{E}_{\text{mech,loss}} = \dot{E}_{\text{mech,loss,pump}} + \dot{E}_{\text{mech,loss,piping}}$  and

$\dot{W}_{\text{pump,u}} = \dot{W}_{\text{pump}} - \dot{E}_{\text{mech,loss,pump}}$ . Noting that  $\dot{E}_{\text{mech,loss,piping}} = \dot{m}gh_L$ , the useful pump power is

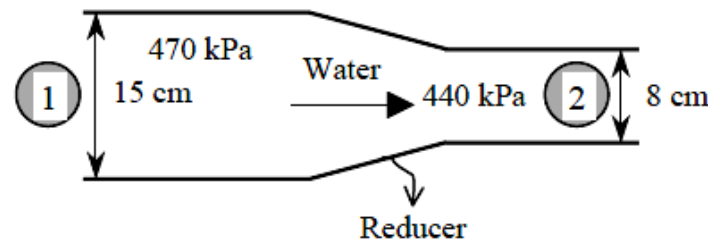
$$\begin{aligned} \dot{W}_{\text{pump,u}} &= \dot{m}gz_2 + \dot{m}gh_L = \rho \dot{V}g(z_2 + h_L) \\ &= (1000 \text{ kg/m}^3)(0.025 \text{ m}^3/\text{s})(9.81 \text{ m/s}^2)[(25 + 7) \text{ m}] \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \\ &= 7.85 \text{ kNm/s} = 7.85 \text{ kW} \end{aligned}$$

Then the mechanical efficiency of the pump becomes

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{pump,u}}}{\dot{W}_{\text{shaft}}} = \frac{7.85 \text{ kW}}{10 \text{ kW}} = 0.785 = \mathbf{78.5\%}$$

**Example 22:** Water flows at a rate of  $0.035 \text{ m}^3/\text{s}$  in a horizontal pipe whose diameter is reduced from 15 cm to 8 cm by a reducer. If the pressure at the centerline is measured to be 470 kPa and 440 kPa before and

after the reducer, respectively, determine the irreversible head loss in the reducer. Take the kinetic energy correction factors to be 1.05.



**Assumptions** 1 The flow is steady and incompressible. 2 The pipe is horizontal. 3 The kinetic energy correction factors are given to be  $\alpha_1 = \alpha_2 = \alpha = 1.05$ .

**Properties** We take the density of water to be  $\rho = 1000 \text{ kg/m}^3$ .

**Analysis** We take points 1 and 2 along the centerline of the pipe before and after the reducer, respectively. Noting that  $z_1 = z_2$ , the energy equation for steady incompressible flow through a control volume between these two points reduces to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \rightarrow h_L = \frac{P_1 - P_2}{\rho g} + \frac{\alpha(V_1^2 - V_2^2)}{2g}$$

where

$$V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\pi D_1^2 / 4} = \frac{0.035 \text{ m}^3/\text{s}}{\pi (0.15 \text{ m})^2 / 4} = 1.98 \text{ m/s}$$

$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2 / 4} = \frac{0.035 \text{ m}^3/\text{s}}{\pi (0.08 \text{ m})^2 / 4} = 6.96 \text{ m/s}$$

Substituting, the head loss in the reducer is determined to be

$$h_L = \frac{(470 - 440) \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kN/m}^2}{1 \text{ kPa}} \right) \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kN}} \right) + \frac{1.05[(1.98 \text{ m/s})^2 - (6.96 \text{ m/s})^2]}{2(9.81 \text{ m/s}^2)} \\ = 3.06 - 2.38 = \mathbf{0.68 \text{ m}}$$

**Example 23:** A hydraulic turbine has 85 m of head available at a flow rate of  $0.25 \text{ m}^3/\text{s}$ , and its overall turbine-generator efficiency is 78 percent. Determine the electric power output of this turbine.

**Assumptions** 1 The flow is steady and incompressible. 2 The available head remains constant.

**Properties** We take the density of water to be  $\rho = 1000 \text{ kg/m}^3$ .

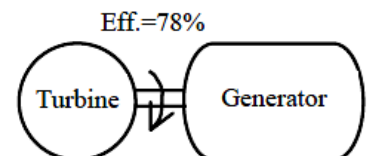
**Analysis** When the turbine head is available, the corresponding power output is determined from

$$\dot{W}_{\text{turbine}} = \eta_{\text{turbine}} \dot{m} g h_{\text{turbine}} = \eta_{\text{turbine}} \rho \dot{V} g h_{\text{turbine}}$$

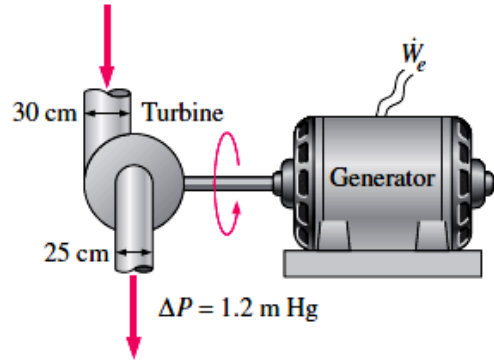
Substituting,

$$\dot{W}_{\text{turbine}} = 0.78(1000 \text{ kg/m}^3)(0.25 \text{ m}^3/\text{s})(9.81 \text{ m/s}^2)(85 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kW}}{1000 \text{ N} \cdot \text{m/s}} \right) = \mathbf{163 \text{ kW}}$$

**Example 24:** Water enters a hydraulic turbine through a 30-cm diameter pipe at a rate of  $0.6 \text{ m}^3/\text{s}$  and exits through a 25-cm diameter pipe. The pressure drop in the turbine is measured by a mercury manometer to be



1.2 m. For a combined turbine– generator efficiency of 83 percent, determine the net electric power output. Disregard the effect of the kinetic energy correction factors.



**Assumptions** 1 The flow is steady and incompressible. 2 All losses in the turbine are accounted for by turbine efficiency and thus  $h_L = 0$ . 3 The elevation difference across the turbine is negligible. 4 The effect of the kinetic energy correction factors is negligible,  $\alpha_1 = \alpha_2 = \alpha = 1$ .

**Properties** We take the density of water to be  $1000 \text{ kg/m}^3$  and the density of mercury to be  $13,560 \text{ kg/m}^3$ .

**Analysis** We choose points 1 and 2 at the inlet and the exit of the turbine, respectively. Noting that the elevation effects are negligible, the energy equation in terms of heads for the turbine reduces to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump},u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine},e} + h_L \rightarrow h_{\text{turbine},e} = \frac{P_1 - P_2}{\rho_{\text{water}} g} + \frac{\alpha(V_1^2 - V_2^2)}{2g} \quad (1)$$

where

$$V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\pi D_1^2 / 4} = \frac{0.6 \text{ m}^3/\text{s}}{\pi (0.30 \text{ m})^2 / 4} = 8.49 \text{ m/s}$$

$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2 / 4} = \frac{0.6 \text{ m}^3/\text{s}}{\pi (0.25 \text{ m})^2 / 4} = 12.2 \text{ m/s}$$

The pressure drop corresponding to a differential height of 1.2 m in the mercury manometer is

$$\begin{aligned} P_1 - P_2 &= (\rho_{\text{Hg}} - \rho_{\text{water}})gh \\ &= [(13,560 - 1000) \text{ kg/m}^3](9.81 \text{ m/s}^2)(1.2 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \\ &= 148 \text{ kN/m}^2 = 148 \text{ kPa} \end{aligned}$$

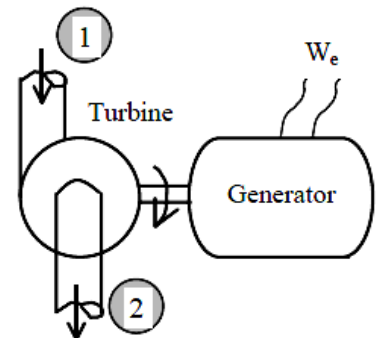
Substituting into Eq. (1), the turbine head is determined to be

$$h_{\text{turbine},e} = \frac{148 \text{ kN/m}^2}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kN}} \right) + (1.0) \frac{(8.49 \text{ m/s})^2 - (12.2 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 15.1 - 3.9 = 11.2 \text{ m}$$

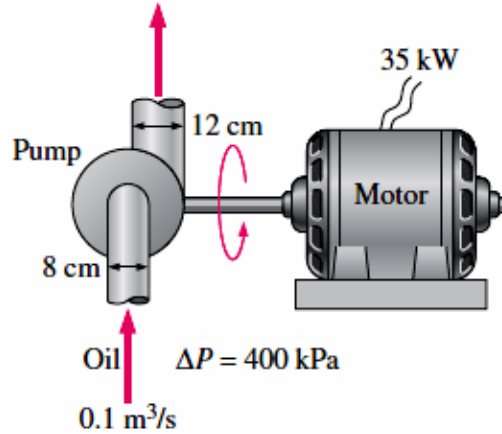
Then the net electric power output of this hydroelectric turbine becomes

$$\begin{aligned} \dot{W}_{\text{turbine}} &= \eta_{\text{turbine-gen}} \dot{m} g h_{\text{turbine},e} = \eta_{\text{turbine-gen}} \rho \dot{V} g h_{\text{turbine},e} \\ &= 0.83(1000 \text{ kg/m}^3)(0.6 \text{ m}^3/\text{s})(9.81 \text{ m/s}^2)(11.2 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kW}}{1000 \text{ N} \cdot \text{m/s}} \right) = 55 \text{ kW} \end{aligned}$$

**Example 25:** An oil pump is drawing 35 kW of electric power while pumping oil with  $\rho = 860 \text{ kg/m}^3$  at a rate of  $0.1 \text{ m}^3/\text{s}$ . The inlet and outlet diameters of the pipe are 8 cm and 12 cm, respectively. If the pressure rise



of oil in the pump is measured to be 400 kPa and the motor efficiency is 90 percent, determine the mechanical efficiency of the pump. Take the kinetic energy correction factor to be 1.05.



**Assumptions** 1 The flow is steady and incompressible. 2 The elevation difference across the pump is negligible. 3 All the losses in the pump are accounted for by the pump efficiency and thus  $h_L = 0$ . 4 The kinetic energy correction factors are given to be  $\alpha_1 = \alpha_2 = \alpha = 1.05$ .

**Properties** The density of oil is given to be  $\rho = 860 \text{ kg/m}^3$ .

**Analysis** We take points 1 and 2 at the inlet and the exit of the pump, respectively. Noting that  $z_1 = z_2$ , the energy equation for the pump reduces to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \rightarrow h_{\text{pump, u}} = \frac{P_2 - P_1}{\rho g} + \frac{\alpha(V_2^2 - V_1^2)}{2g}$$

where

$$V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\pi D_1^2 / 4} = \frac{0.1 \text{ m}^3/\text{s}}{\pi (0.08 \text{ m})^2 / 4} = 19.9 \text{ m/s}$$

$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2 / 4} = \frac{0.1 \text{ m}^3/\text{s}}{\pi (0.12 \text{ m})^2 / 4} = 8.84 \text{ m/s}$$

Substituting, the useful pump head and the corresponding useful pumping power are determined to be

$$h_{\text{pump, u}} = \frac{400,000 \text{ N/m}^2}{(860 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) + \frac{1.05[(8.84 \text{ m/s})^2 - (19.9 \text{ m/s})^2]}{2(9.81 \text{ m/s}^2)} = 47.4 - 17.0 = 30.4 \text{ m}$$

$$\dot{W}_{\text{pump, u}} = \rho \dot{V} g h_{\text{pump, u}} = (860 \text{ kg/m}^3)(0.1 \text{ m}^3/\text{s})(9.81 \text{ m/s}^2)(30.4 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kW}}{1 \text{ kN} \cdot \text{m/s}} \right) = 25.6 \text{ kW}$$

Then the shaft pumping power and the mechanical efficiency of the pump become

$$\dot{W}_{\text{pump, shaft}} = \eta_{\text{motor}} \dot{W}_{\text{electric}} = (0.90)(35 \text{ kW}) = 31.5 \text{ kW}$$

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{pump, shaft}}} = \frac{25.6 \text{ kW}}{31.5 \text{ kW}} = 0.813 = 81.3\%$$

